The meaning of superfluidity for polariton condensates

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Some milestones of liquid He superfluidity

Liquid $^4$He below $\Lambda$ point (2.2 K):

- $\Lambda$ singularity in thermodynamic functions

- Flows through narrow capillars with no viscosity
  Kapitsa; Allen and Misener (1937)

- Rotating bucket experiment: lattice of quantized vortices
  Figure from: Yarmchuk, Gordon and Packard PRL (1979)

- Fountain effect: superfluid $^4$He climbs over container walls when heated
Several definitions of superfluidity

1 - Rotating bucket experiment, i.e. equilibrium in rotating frame:
- no rotation of fluid for $\Omega < \Omega_c$ (Hess-Fairbank effect)
- lattice of quantized vortices for $\Omega > \Omega_c$

2 - Meissner-like effect:
- absence of response to transverse gauge field, i.e. transverse current response
- normal, non-superfluid fraction defined as:
  $$\lim_{q \to 0} X^T(q, \omega = 0) = \frac{N}{m} \frac{\rho_n}{\rho}$$

3 - Metastability of superflow:
- flow persists for macroscopic times even in the presence of defects

4 - Landau criterion:
- no drag force on object slowly moving through the fluid
- Landau critical velocity:
  $$v_c = \min_k \frac{\omega(k)}{k}$$


Superfluidity in equilibrium Bose systems

<table>
<thead>
<tr>
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<th>Non-interacting BEC</th>
<th>Interacting BEC</th>
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<tbody>
<tr>
<td>Rotating bucket experiment</td>
<td>YES</td>
<td>YES</td>
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<tr>
<td>Transverse current response</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Persistent currents</td>
<td>NO</td>
<td>YES</td>
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<tr>
<td>No drag force</td>
<td>NO</td>
<td>YES</td>
</tr>
</tbody>
</table>

Interacting BEC : all definitions coincide!
Recent experiments with atomic BECs (I)

$v=0.35 \text{ mm/s} < v_c$

$v=5.6 \text{ mm/s} > v_c$

**moving defect:**

$v < v_c$ : dissipationless flow

$v > v_c$ : asymmetry in density profile

Figure from: R. Onofrio et al. PRL 85, 2228 (2000)

Lattice of quantized vortices in stirred BEC

Figure from: Abo-Shaeer et al., Science 292, 476 (2001)
Recent experiments with atomic BECs (II)

Experimental picture taken at JILA, courtesy E. Cornell and P. Engels

Recent experiments with atomic BECs (III)

Metastability of supercurrents in ring traps during macroscopic times

Images from:
C. Ryu, M. F. Andersen, P. Cladé, Vasant Natarajan, K. Helmerson, and W. D. Phillips
PRL 99, 260401 (2007)
**What to expect in non-equilibrium systems**

1. Metastability of superflow → YES

**Gyrolaser** for aeronautics

exploit a kind of metastable supercurrent of light
What to expect in non-equilibrium systems

2 - Quantized vortices → YES

Vortices can spontaneously appear even in the absence of stirring

Single-valued macroscopic wf. implies quantized winding number

Does NOT prove superfluidity

More details in talks by Wouters, Lagoudakis and Keeling

Figure taken from: K. G. Lagoudakis, M. Wouters, M. Richard, A. Baas, IC, R. André, Le Si Dang, B. Deveaud-Pledran, Quantised Vortices in an Exciton Polariton Fluid, preprint arXiv/0801.1916, Nature Physics (in the press)
What to expect in non-equilibrium systems

3 – Response to transverse gauge field

- Minimal coupling Hamiltonian
  \[ H = \frac{[P - e A(r)]^2}{2m} \]
- Gauge field
  \[ A(r) = A_0 e^{i q \cdot r} + c.c. \]
- Transverse condition
  \[ q \cdot A = 0 \]
- Current-current response
  \[ J(q, \omega) = \chi(q, \omega) A(q, \omega) \]
- Normal, non-superfluid fraction related to transverse response:
  \[ \lim_{q \to 0} \chi^T(q, \omega = 0) = \frac{N}{m} \frac{\rho_n}{\rho} \]

But...

- How to generate the A field acting on (neutral) polaritons?
- For atoms: topological potentials in 3-level EIT-like configurations
What to expect in non-equilibrium systems

4 - Drag force depends on pumping scheme

Landau criterion for superfluidity:

- Drag force $F(v)$ on object moving through the quantum fluid
- No drag force for slow objects $F(v < v_c) = 0$
- Landau critical velocity: $v_c = \min_k \frac{\omega(k)}{k}$

- Coherent polaritons injected by resonant pump
  (IC, Ciuti, PRL 2004)

- OPO system (Madrid expts)

- Polariton BEC under non-resonant pump

Landau predicts non-superfluid behaviour.
But crossover still visible in $F(v)$

Complex physics, see talks by Sanvitto, Del Valle and Laussy.
Exciton-polaritons in DBR microcavity with QWs

- **DBR:** stack $\lambda/4$ layers (e.g. GaAs/AlAs)
- Cavity layer $\rightarrow$ confined photonic mode, delocalized along 2D plane:
  \[ \omega_C(k) = \omega^0_C \sqrt{1 + \frac{k^2}{k_z^2}} \]
- e and h confined in QW layer (e.g. InGaAs)
- e-h pair: sort of H atom. **Exciton**
- **Excitons bosons** if $n_{\text{exc}} a_{\text{Bohr}}^2 \ll 1$
- **Excitons delocalized** along cavity plane.
  
  Flat exciton dispersion $\omega_x(k) \approx \omega_x$

Radiative coupling between excitonic transition and cavity photon at same in-plane $k$

**Eigenmodes:** bosonic superpositions of exciton and photon, called **polaritons**
Polariton BEC under non-resonant pumping

- high-energy pump creates incoherent e-h pairs
- relaxation via polariton-polariton collisions, quasi-equilibrium condition
- at high pumping, stimulated scattering into low-energy modes
- coherence spontaneously created via BEC effect

Elementary excitations of polariton BEC

- system far from thermodynamical equilibrium
- dynamical balance of pump and losses
- $+$ mode satisfies Goldstone theorem
- but has diffusive nature at low $k$
  $$\text{Re}[\omega(k)] = 0, \quad \text{Im}[\omega(k)] = -a k^2$$
- recovers Bogoliubov sound at high $k$

Consequences on superfluidity of polariton BECs

Naïf Landau argument:

- Landau critical velocity $v_c = \min_k \frac{\omega(k)}{k} = 0$ for non-equilibrium BEC
- Any moving defect expected to emit phonons
But non-equilibrium life is bit more complicate...

- **drag force** $F(v)$:
  - $\Gamma / \mu = 0$: recovers Landau criterion, $F(v<v_c)=0$
  - $\Gamma / \mu > 0$: smoothened, still crossover behaviour

- **real-space “Cerenkov” wake**:
  - localized perturbation for small $v$
  - propagating phonons for large $v$
Generalized Landau criterion with complex k's

Low $v$:
- emitted $k_\parallel$ purely imaginary
- no real propagating phonons
- localized perturbation around defect

Critical velocity $v < c$:
- corresponds to bifurcation point
- decreases with $\frac{\Gamma}{\mu}$

High $v$:
- propagating phonons are emitted:
  - Cerenkov cone
  - parabolic precursors
- spatial damping of Cerenkov cone

Conclusions and outlook

A new frontier:

**superfluidity effects in non-equilibrium systems**

Non-equilibrium Landau superfluidity very rich world:

- under coherent pump: from standard Cerenkov to zebra-Cerenkov

- Non-resonantly pumped BECs:
  - → diffusive Goldstone mode
  - → zero Landau critical velocity, but remnants of superfluidity apparent

Other aspects of polariton superfluidity that remain to be explored:

- metastability of supercurrents, possible gyroscopic sensor applications
- transverse current-current fluctuations and response to gauge fields
Thanks to my brave coworkers...

Michiel Wouters

Cristiano Ciuti

Simon Pigeon
Experimental data: linear regime

Polariton cloud expanding against a defect

Real space pattern: fringes

k-space pattern
Resonant Rayleigh scattering ring

Figure from Houdré et al, PRL (2000)

Figure from Langbein (2002)
Madrid experiment: a different regime

More complex OPO configuration:

- coherent pump, parametric oscillation into signal and idler modes
- defect acts on all of them
- wavepacket of signal polaritons moving against defect

A. Amo et al., arxiv/0711.1539

Weakly perturbed probe: superfluid? Fringes: non-superfluid pump
Basics of the OPO regime

- **CW pump** at \(k_p\) close to “magic angle” condition \(2\omega(k_p) = \omega(k_s) + \omega(k_i)\)
- **Parametric oscillation**: ever lasting coherent emission at \(k_{s,i}\)
- No need for seeding \(k_{s,i}\), zero point fluctuations enough to start process
- **Short pulse** into \(k_{s,i}\) may serve to force mode selection
Small fluctuations around OPO state

Steady-state above threshold:

- coherent signal/idler beams
- $U(1)$ symmetry spontaneously broken
- soft Goldstone mode $\omega_G(k) \to 0$ for $k \to 0$
- corresponds to slow signal-idler phase rotation
  → as Bogoliubov phonon at equilibrium !!!

Fundamental physical difference:

→ Goldstone mode diffusive, not propagating like sound

Consequences for superfluidity of an OPO state

Diffusive Goldstone mode

Naïve application of Landau's argument:
- excitations created by the defect for low speed
- no superfluidity properties

Complete calculations is in progress
- rich excitation spectrum
- pump and signal/idler modes to be identified
- all of them excited by the defect
Scattering on defect: far-field emission patterns

Linear regime:
- peak at $k_p$ due to unscattered light
- resonant Rayleigh scattering ring

Super-sonic flow $v_p > c_s$:
- ∞ shaped pattern
- two lobes touch at $k_p$
- left lobe (particle) stronger than right lobe (hole)

Sub-sonic flow $v_p < c_s$:
- Landau criterion predicts superfluidity
- lobes disappear, much weaker scattering on defect
Near-field emission patterns

Linear regime:
- interference of incident and scattered field gives parabolic wavefronts

Super-sonic flow $v_p > c_s$:
- sound waves form Cerenkov cone
- aperture: $\sin \theta = c_s / v_p$
- parabolic precursors upstream

Subsonic flow $v_p < c_s$:
- localized perturbation
- superfluid flow around defect
**Polaritons much richer than atomic BECs**

Richer phenomenology as oscillation freq. freely fixed by $\mu = \omega_p$

**An example:**

- **Concentric rings**: usual RRS ring
- **Zebra pattern**: narrow k-space peaks, precursor of parametric oscillation
- **But many other shapes** possible by tuning angle, frequency, intensity ...

Coherently driven microcavity

Plane wave monochromatic pump
- fixes condensate $k=k_p$, finite drift velocity $v_p$
- oscillation frequency $\mu$ selected by $\omega_p$,
  not by Eq. of state $\mu=gn$ as in equilibrium BECs
- Rich density vs. pump intensity dependence

$\omega_p < \omega_{LP}(k_p)$
optical limiting

$\omega_p > \omega_{LP}(k_p)$
optical bistability
Resonant point: same behaviour as in atomic BEC

Gapless spectrum of Bogoliubov modes around pump-only state: tilting by $v_p$

For growing interactions $\delta \omega_{\text{MF}} = g |\psi_{X}^{\text{ss}}|^2$:

- almost parabolic dispersion for small $\delta \omega_{\text{MF}}$
- sound velocity increases with $\delta \omega_{\text{MF}}$
- sound velocity eventually changes sign and intersection with $E-E_p=0$ disappears (Landau criterion of superfluidity)