Neutral Fermions and Skyrmions in the Moore-Read state at $\nu = 5/2$

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Royal Holloway, March 15, 2012

Overview

Introduction

- Fractional Quantum Hall effect (QHE) and the story of $\nu = 5/2$

Neutral fermion excitations in $\nu = 5/2$

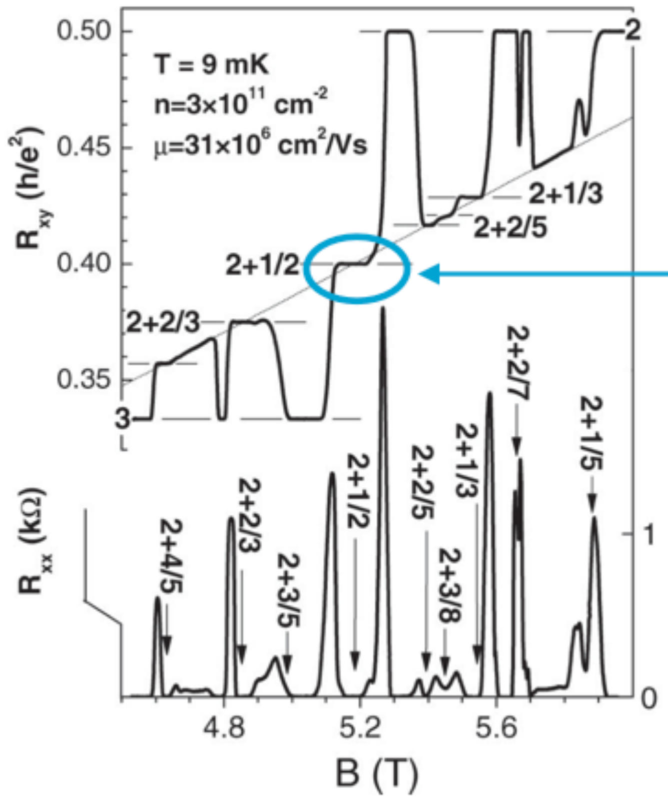
- Neutral Fermions: qualitative features of pairing physics and non-abelian statistics

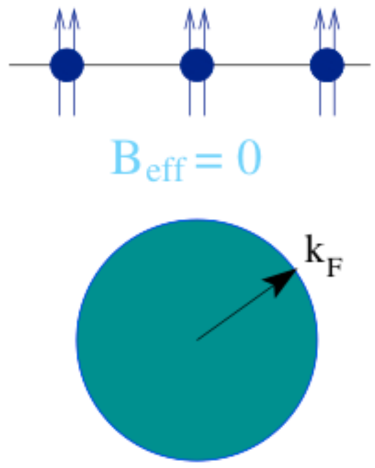
Photoluminescence experiments at $\nu = 5/2$

- Characteristic features of the PL spectrum

- Conclusions

The QHE at $\nu = 5/2$: a one-slide introduction





$$\Psi = \mathcal{P}_{\rm LLL} \prod_{i<j} (z_i - z_j)^2 \psi^{\rm CF}(\{z_i\})$$

proposition: p-wave pairing in LL2

$$\Psi_{MR} = \prod_{i<j} (z_i - z_j)^2 \mathrm{Pf}\left[\frac{1}{z_i - z_j}\right]$$

can optimize pair wavefunction without changing topological order

The Moore Read wavefunction

NONABELIONS IN THE FRACTIONAL QUANTUM HALL EFFECT

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Received 31 May 1990
(Revised 5 December 1990)

$$\Psi_{MR} = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right)\prod_{i<j}(z_i - z_j)^2$$ = Paired chiral p-wave composite fermions

Since the combination $\psi^\dagger U^q$ is always a fermion at $\nu = 1/q$, q even, and so these must pair if they are to have any chance to condense, and since the pfaffian state is the simplest way for them to do so, we feel that it is likely that if an incompressible state is ever observed at these filling factors with full spin polarization, it should be this state. Such a state will inevitably have neutral fermion and charged nonabelion excitations.

Nature of quasiparticles

$e/4$ quasiparticles $\leftrightarrow$ vortices of a p-wave SC

- directly probing non-abelian statistics difficult
- considerable overlaps with trial states [e.g. works by Morf, Wójs]
- qp size large compared to system size for numerical calculations
- occur in pairs $\Rightarrow$ more finite size effects

Neutral fermion (NF) $\leftrightarrow$ Bogoliubov quasiparticles

Bogoliubov theory for p-wave SF: $|\mathbf{k}\rangle = \gamma^\dagger_{\mathbf{k}} |\mathrm{BCS}\rangle$,

with $E_{\mathbf{k}} = \sqrt{\frac{1}{2m^*}(k^2 - k_F^2)^2 + k^2\Delta^2}$, and $\gamma_{\mathbf{k}} = u^*_{\mathbf{k}}\hat{c}_{\mathbf{k}} + v_{\mathbf{k}}\hat{c}^\dagger_{\mathbf{k}}$.

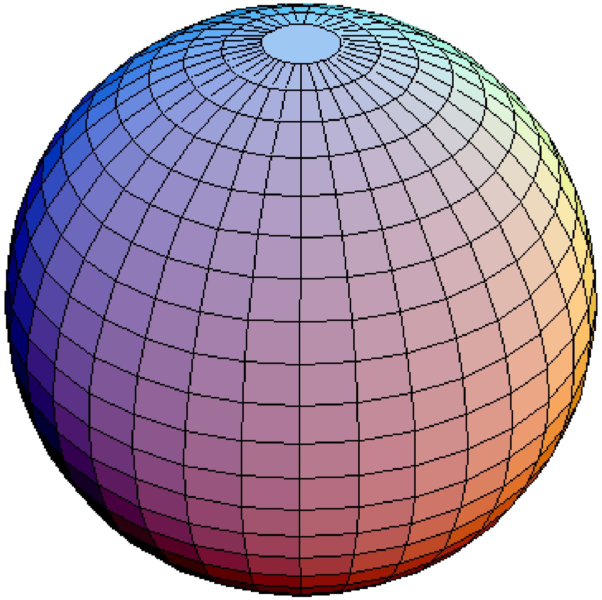
- single localized quasiparticle
- called ‘neutral’, as addition of $1e^-$ and 2 flux quanta conserves overall charge density ρ of ground state
- *pair-breakers* – NF gap direct evidence for pairing in the system

Numerical studies on the sphere

Our tool: exact diagonalization on the sphere

- Convenient geometry without boundaries
- Shift σ relating integer number of flux N_ϕ and number of particles N naturally separates Hilbert-spaces of competing states

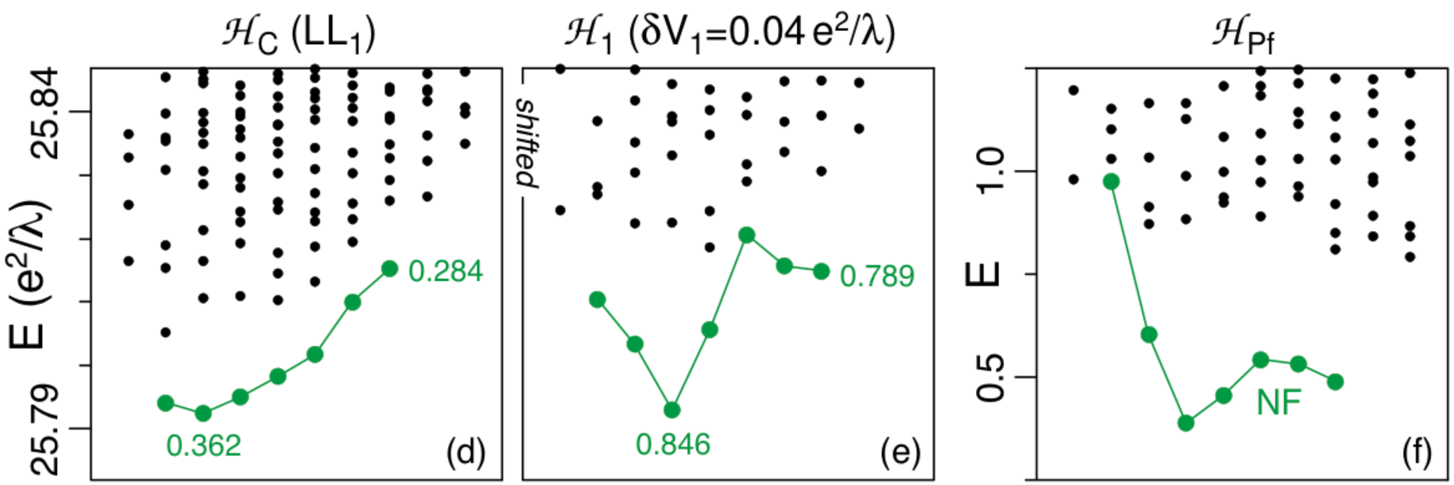
$$N_\phi = \nu^{-1} N - \sigma$$



Diagonalize Hamiltonian in subspace with fixed quantum numbers L, L_z, $[S, S_z]$, using a *projected* Lanczos algorithm.

Numerical studies of $\nu = 5/2$ on the sphere
Sample spectra

Angular-momentum resolved spectra for different Hamiltonians (Coulomb, modified Coulomb, Pfaffian model $\mathcal{H}_{\mathrm{Pf}} = \sum P_{ijk}^{(m=3)}$ at the shift of the Moore-Read state $N_\phi = 2N - 3$, with odd $N(= 15)$



(d) Coulomb Hamiltonian $\mathcal{H}_C$, (e) $\mathcal{H}_1 = \mathcal{H}_C + 0.04\hat{V}_1$, (f) Three-body repulsion $\mathcal{H}_{\mathrm{Pf}}$

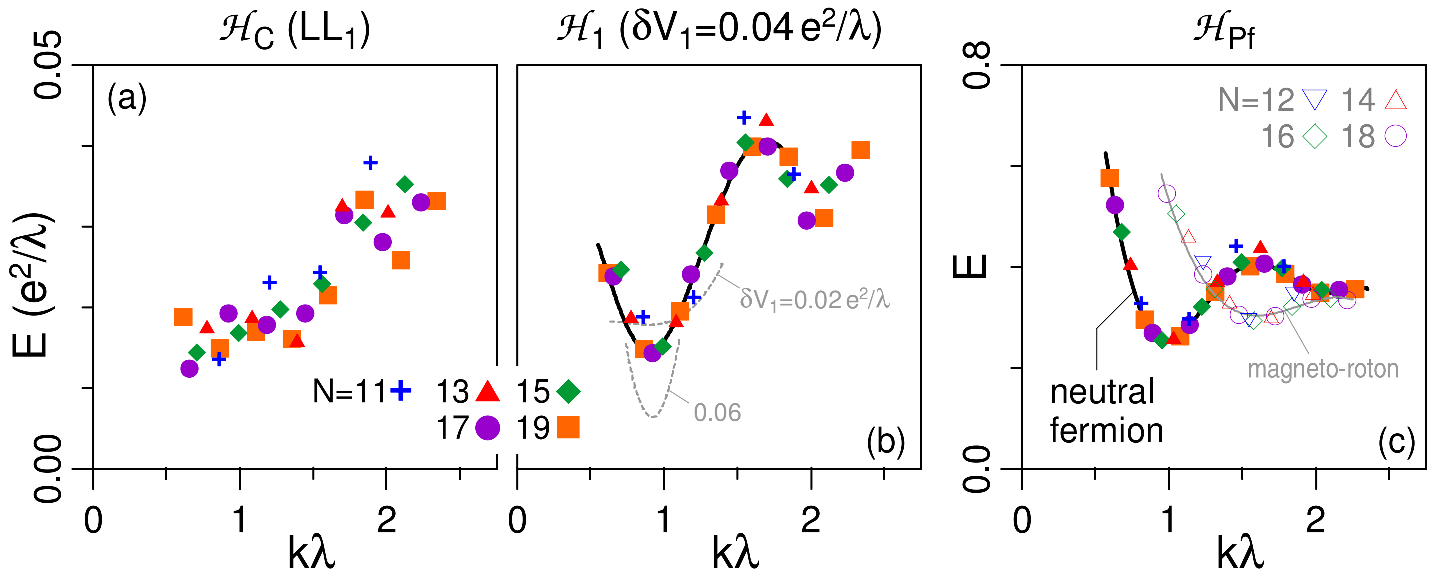
- dispersive mode well separated from the continuum
- spacing of levels $\Delta L = 1 \Rightarrow$ single particle

GM, A. Wójs, and N. R. Cooper, Phys. Rev. Lett. **107**, 036803 (2011)

Numerical studies of $\nu = 5/2$ on the sphere

Dispersion of the neutral fermion mode

Dispersion relation from spectra of $N = 11, \ldots, 19$
[shifted to account for finite-size scaling of $E_0(N) \simeq \Delta_{\mathrm{NF}} + \beta/N$]

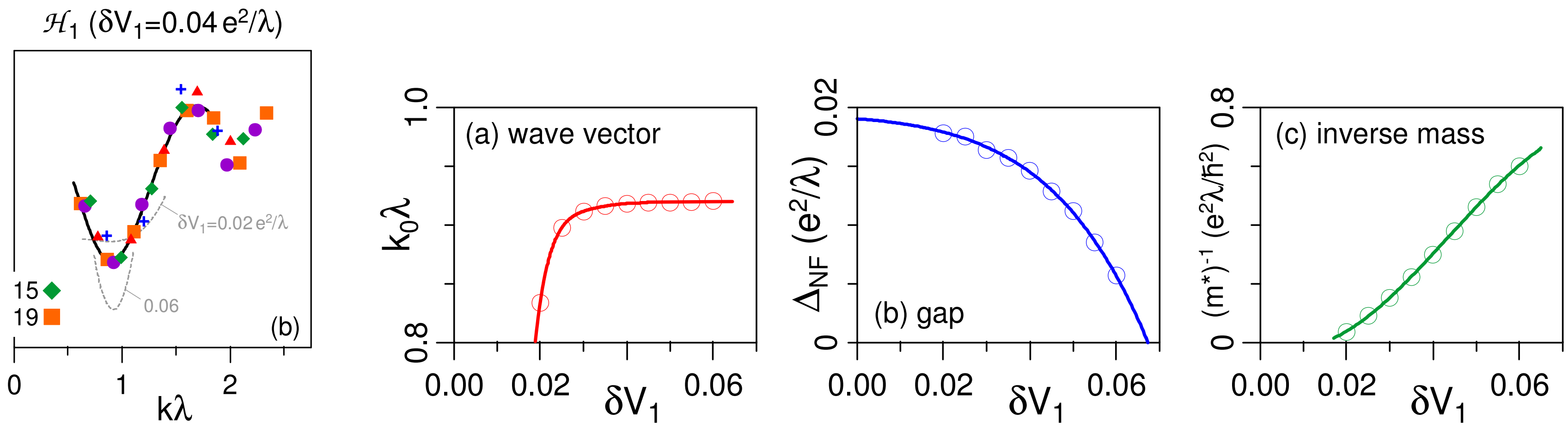


- well formed dispersion for $\delta V_1 > 0$ (or LL-mixing) has *two minima* ($\rightsquigarrow$ phase transition near $\mathcal{H}_C$, Rezayi & Haldane '00)
- second minimum sharp feature (below NF+MR threshold)
- finite gap Δ_{NF} [see also Bonderson et al. PRL '11]
- qualitative features of Pfaffian-model reproduced

Numerical studies of $\nu = 5/2$ on the sphere

Evolution of NF Dispersion – parameters near minimum of dispersion

Tune interactions from 2nd LL-like ($\delta V_1 = 0$) to LLL-like ($\delta V_1 \simeq 0.08$)

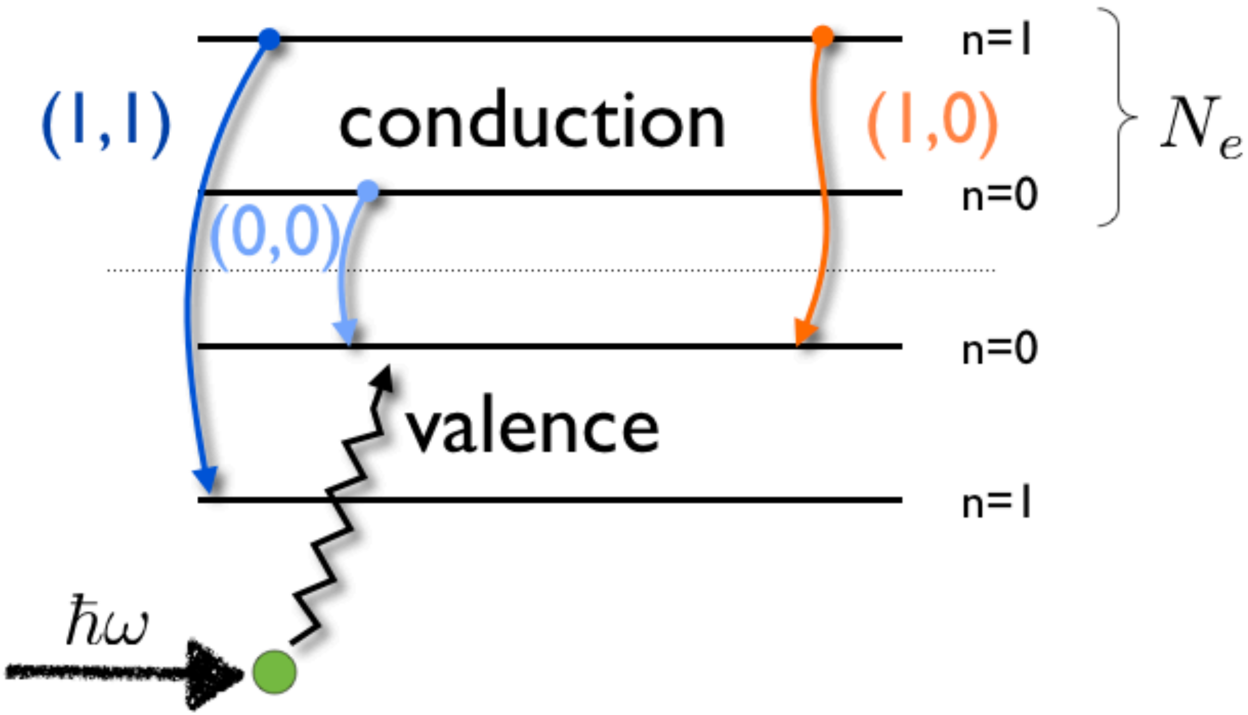


- minimum of dispersion near Fermi-momentum $k_0 \sim k_F = \lambda$
  $\Rightarrow$ evidence of Fermi surface: k_F signature of spin polarization
- Δ_{NF} remains finite at small δV_1 – first order transition to CDW
- Δ_{NF} collapses gradually at large δV_1, while effective NF mass diverges (BdG $\to$ kink!)

Experimental signature of NF Dispersion

How to probe NF dispersion? – Need to change (electron-) fermion #.

Photoluminescence (PL) is a suitable probe (ignoring role of spin below):

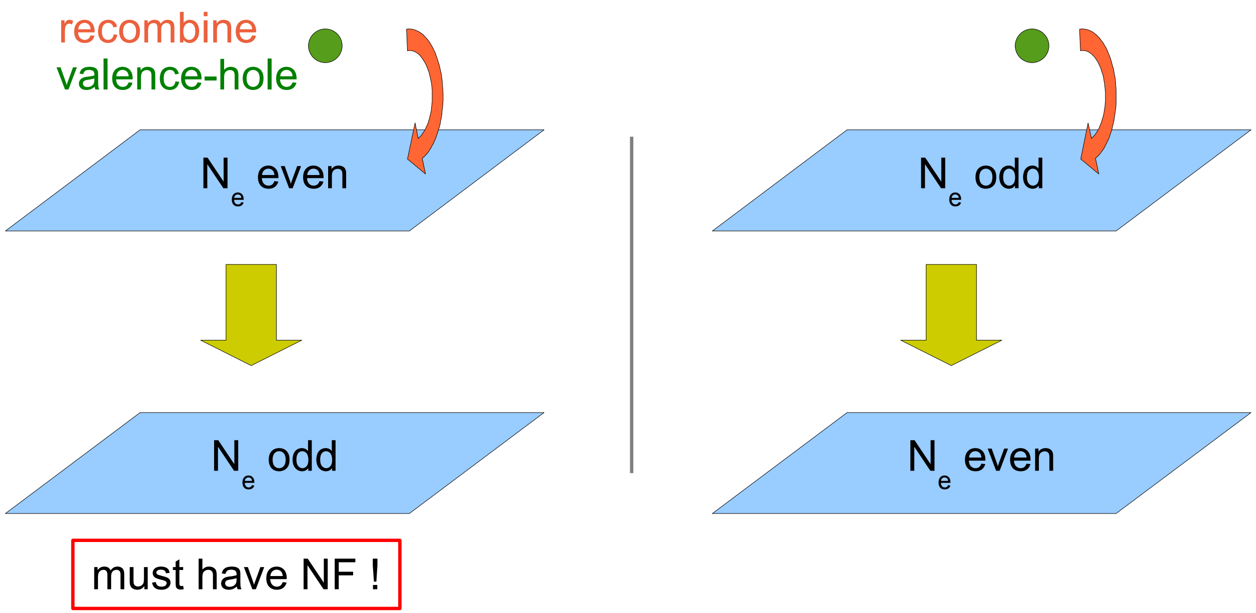


- valence hole h^+ relaxes thermally and then recombines with carriers in 2DEG
- need non-zero matrix-element with 2nd LL electrons

Experimental signature of NF Dispersion

How to probe NF dispersion? – Need to change (electron-) fermion #.

Photoluminescence (PL) is a suitable probe: 2 possible processes

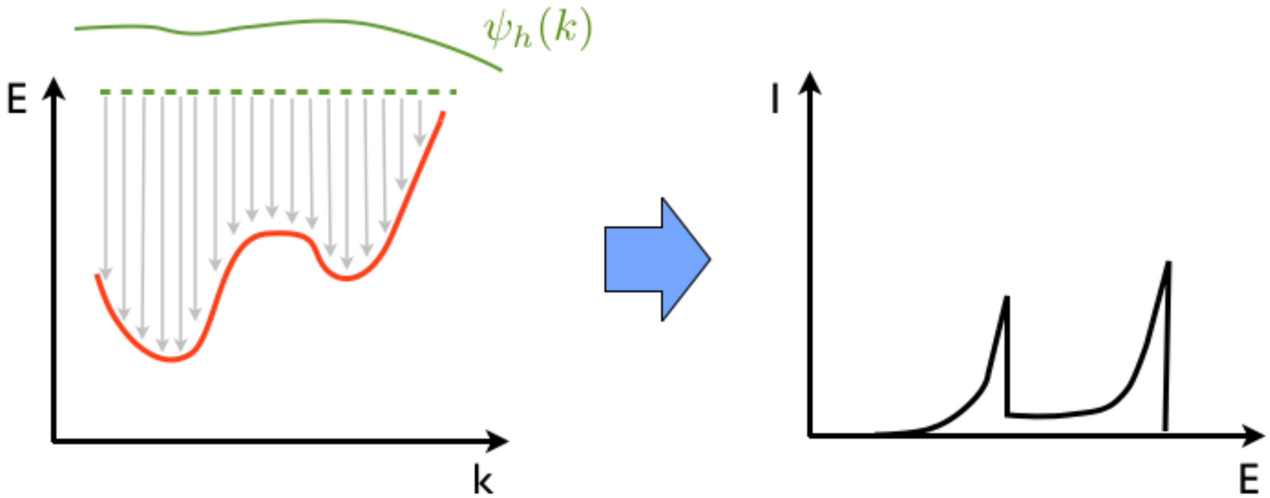


- *initial* state: even N is preferred in ground state (disorder?)
- any *final* state with odd N entails presence of a NF

Experimental signature of NF Dispersion

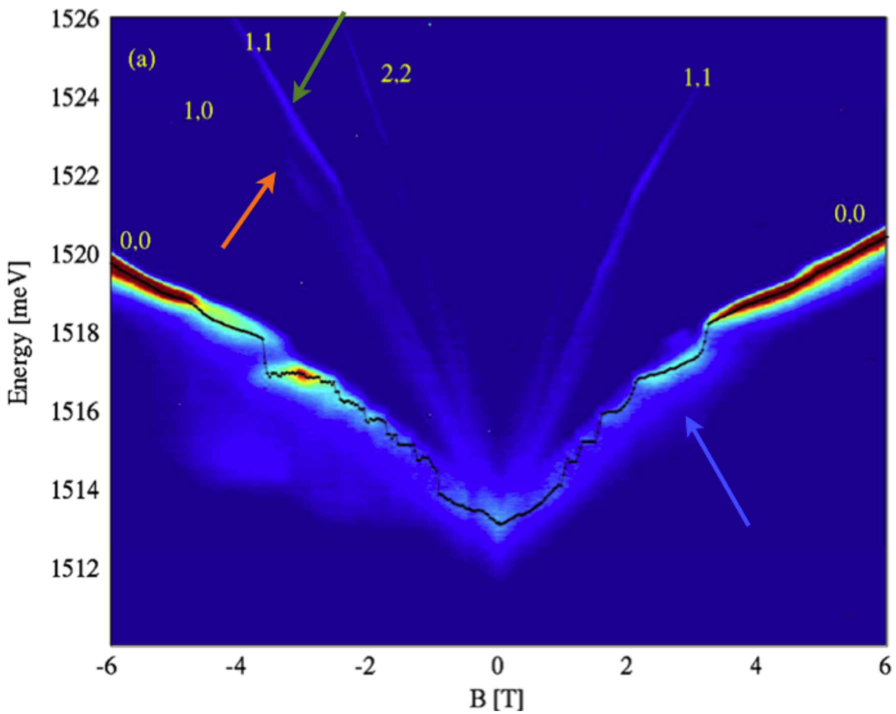
What does one see in PL experiments of the Moore-Read state?

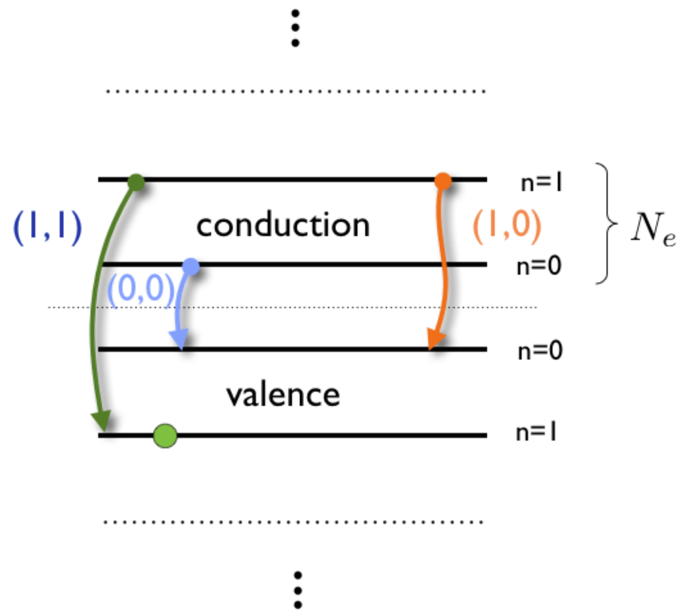
localized h^+ essentially probes DOS $\Rightarrow$ double-peak structure in PL of (1,0) or (1,1) transitions



- each of the threshold peaks may have 'shake-up' processes involving additional magnetorotons

PL experiments in practice
Signals for recombination in different channels





Direct recombination in 2nd LL visible experimentally
(albeit weaker than LLL⇒LLL)

M. Stern *et al.*, Phys. Rev. Lett. (2010), J. K. Jain, Physics (2010)

Energetics of the neutral fermion in presence of quasiholes

identification of fusion channels

- 2QH with even N: 1 channel
- 2QH with odd N (neutral fermion present): ψ channel

study of energetics

- well separated QH: topological degeneracy of fusion channels
- QH in proximity of each other: splitting favours ψ-channel

(ask to see details)

GM, A. Wójs, and N. R. Cooper, Phys. Rev. Lett. **107**, 036803 (2011)

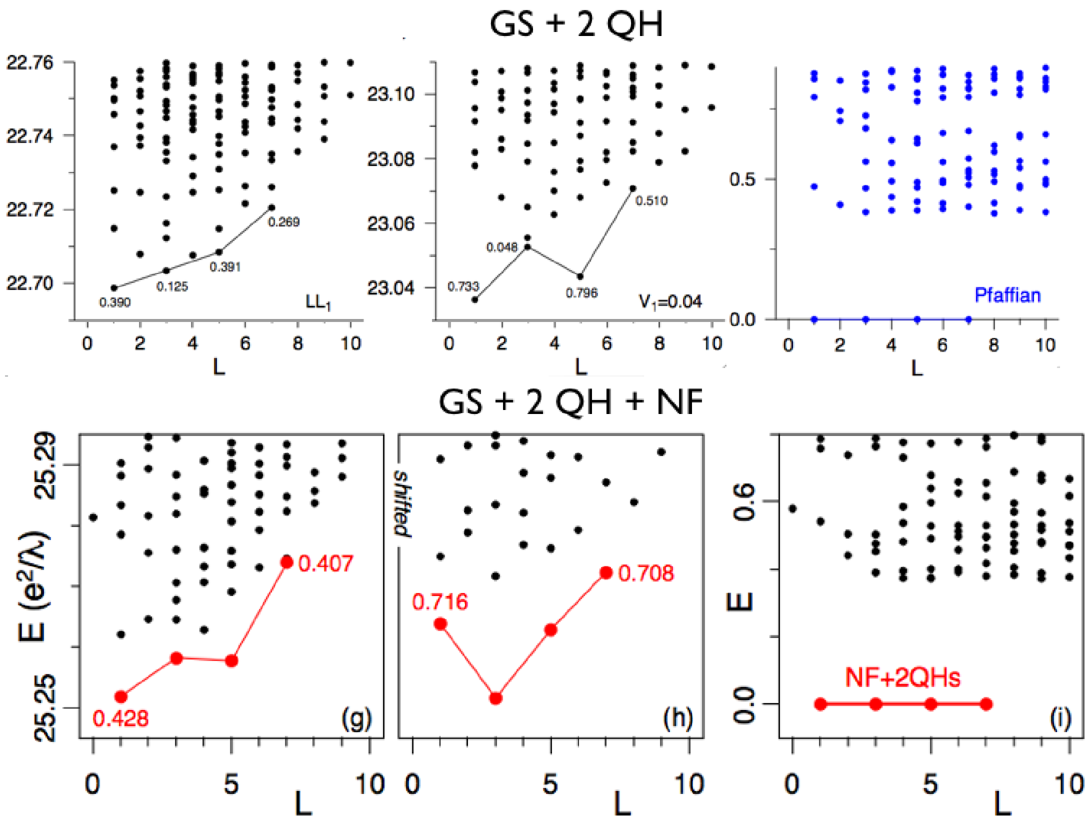
Conclusions

- Presence of neutral fermion excitations affirms the pairing character of the $\nu = 5/2$ state without referring to trial wavefunctions
- Characteristic structure of NF dispersion with double minimum observable both qualitatively and quantitatively in photoluminescence (PL)
- Energetics consistent with topologically degenerate fusion channels 1, ψ of QPs
- First determination of the splitting of fusion channels for both QHs and QEs

GM, A. Wójs, and N. R. Cooper, Phys. Rev. Lett. **107**, 036803 (2011)

Energetics of the NF in presence of quasiparticles

Spectra of quasihole states (insertion of one flux quantum to the GS)



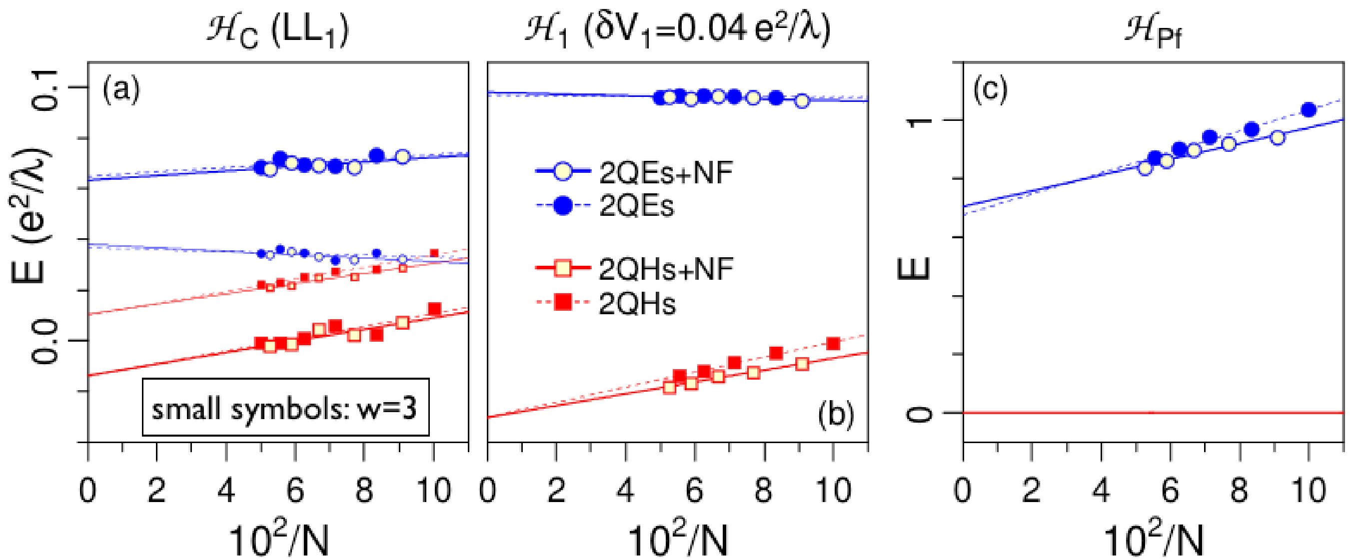
- Spacing of angular momenta $\Delta L = 2$ indicates pair of mobile quasiparticles
- Dispersive band of low-energy excitations both in presence and absence of NF $\Rightarrow$ tricky to compare energies

Energetics of the NF in presence of quasiparticles

In presence of QPs: parity of fermion # $\leftrightarrow$ fusion-channel 1 or ψ

Probe energies of excited states (2QP [+NF]) relative to homogeneous groundstate.

- case 1: arbitrary position of QHs – average within low-lying band



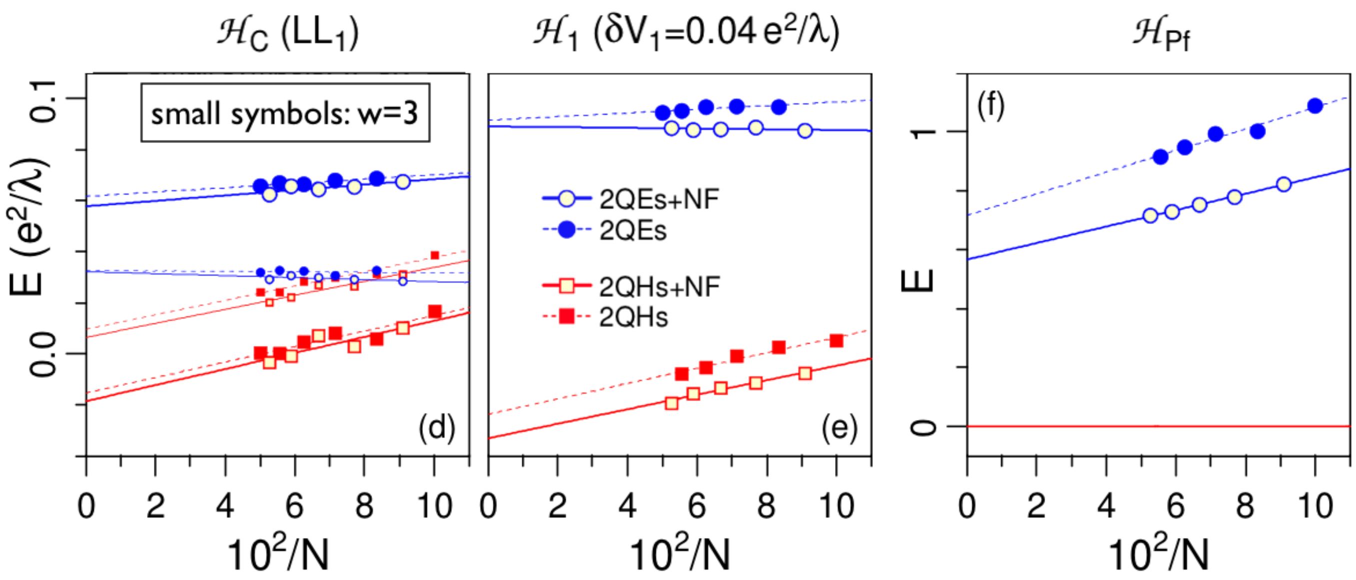
$\Rightarrow$ fusion-channels degenerate for well-separated QPs

Energetics of the NF in presence of quasiparticles

In presence of QPs: parity of fermion # $\leftrightarrow$ fusion-channel 1 or ψ

Probe energies of excited states (2QP [+NF]) relative to homogeneous groundstate.

- case 2: QP's nearby – largest angular momentum



$\Rightarrow$ ψ-channel wins at small r, for both QE and QH

Conclusions

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