Adiabatic connection of Fractional Chern Insulators to Fractional Quantum Hall States

Gunnar Möller
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TCM

GM & N. R. Cooper, Phys. Rev. Lett. **103**, 105303 (2009)
Th. Scaffidi & GM, Phys. Rev. Lett. **109**, 246805 (2012)
A. Sterdyniak, N. Regnault, & GM, Phys. Rev. B **86**, 165314 (2012)

November 21, 2012, University of Leeds

Trinity Hall Cambridge
The Leverhulme Trust
Cavendish Laboratory
University of Cambridge

Outline

- Motivation: physical realizations of lattices with high flux density

- Relationship between different realizations of topological band structures:
  - Adiabatic continuation from Fractional Chern Insulators to Fractional Quantum Hall states

- Particle entanglement spectrum as a probe for quantum Hall states in lattices

Magnetic flux through periodic potentials

- magnetic field in presence of crystal potentials in solid state systems

$$\ell_0 \gg a \qquad n \simeq 1$$

✗

Magnetic flux through periodic potentials

- magnetic field in presence of crystal potentials in solid state systems $\ell_0 \gg a$ $n \simeq 1$ ✗

- cold atoms in `rotating' optical lattices: $\ell_0 \sim a$ $n \gg 1$ ✗

- Simulating Aharonov-Bohm flux by complex hopping in tight binding optical lattices $\ell_0 \simeq a$ $n \simeq 1$

- Simulating Aharonov-Bohm flux by Berry phases in real space

  ➡ Simulation of Landau-gauge (continuum) $\ell_0 \sim L^{1/2}$

  ➡ "Optical Flux Lattices" $\ell_0 \simeq a$ $n \simeq 1$

Cold Atoms

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Cold Atoms

- Chern bands: Berry flux in reciprocal space

$\ell_0 \simeq a$ $n \simeq 1$

Cold Atoms

Solid State

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Cold Atoms

- Chern bands: Berry flux in reciprocal space

$\ell_0 \simeq a$ $n \simeq 1$

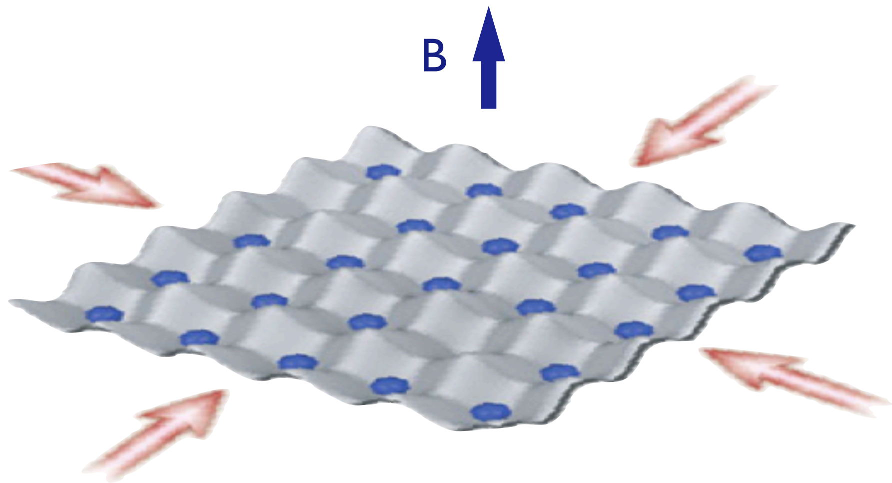
Cold Atoms

Solid State

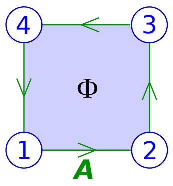
Cold Atoms I: Optical Lattices with Complex Hopping

experimental realisation: optical lattice + Raman lasers

⇒ possibility to simulate Aharonov-Bohm effect of magnetic field by imprinting phases for hopping via Raman transitions



$$\sum_{\square} A_{\alpha\beta} = 2\pi n_{\rm V}$$



⇒ Bose-Hubbard with a magnetic field (→ Lorentz force)

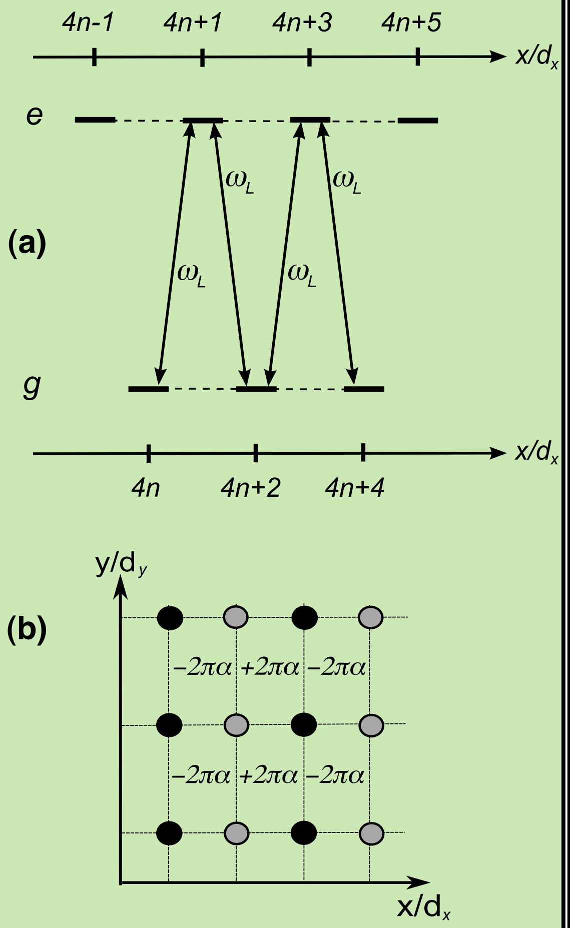
$$\mathcal{H} = -J \sum_{\langle\alpha,\beta\rangle} \left[\hat{b}^\dagger_\alpha \hat{b}_\beta e^{iA_{\alpha\beta}} + h.c.\right] + \frac{1}{2} U \sum_\alpha \hat{n}_\alpha(\hat{n}_\alpha - 1) - \mu \sum_\alpha \hat{n}_\alpha$$

particle density n vortex/flux density $n_{\rm V}$ interaction U/J

J. Dalibard, et al. Rev. Mod. Phys. 83, 1523 (2011)

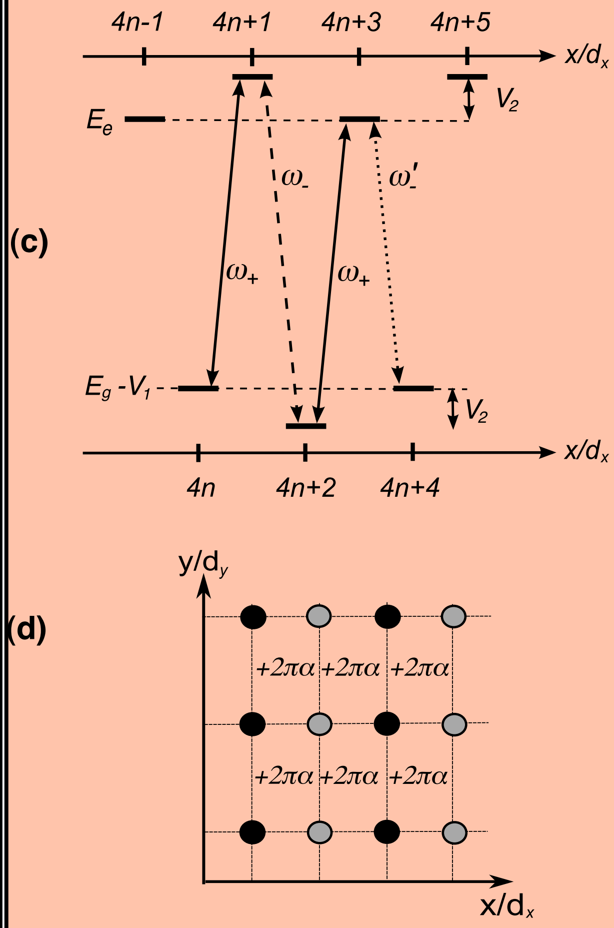
Cold Atoms I: Optical Lattices with Complex Hopping

Staggered Flux



Realized by group of I. Bloch:
Phys. Rev. Lett. **107**, 255301 (2011).

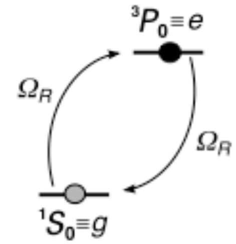
'Rectified' Flux

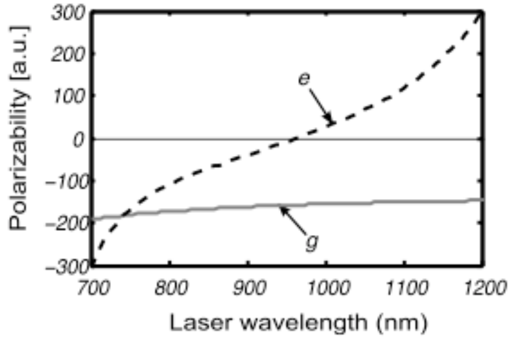


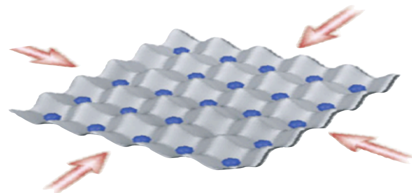
Can tune flux density!

Experimental realization outstanding

'anti-magic' optical lattice for Yb





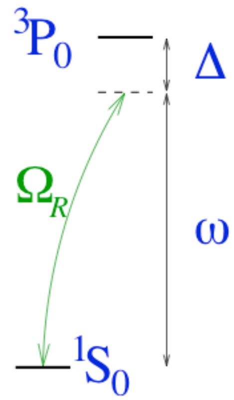


Gerbier & Dalibard, NJP 2010

Cold Atoms II: Berry phases of optically dressed states

J. Dalibard, F. Gerbier, G. Juzeliunas, P. Öhberg, RMP 2011

spacially varying optical coupling of *N* internal states (consider N=2)



Yb

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m}\hat{1} + \frac{\hbar}{2}\begin{pmatrix} -\Delta & \Omega_R(\mathbf{r}) \\ \Omega_R(\mathbf{r}) & \Delta \end{pmatrix}$$

local spectrum $E_n(\mathbf{r})$ and dressed states: $|\Psi_\mathbf{r}\rangle$

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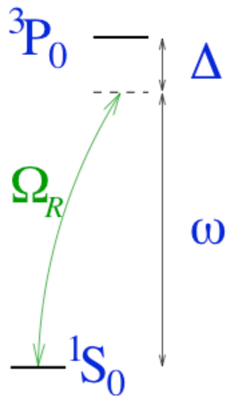

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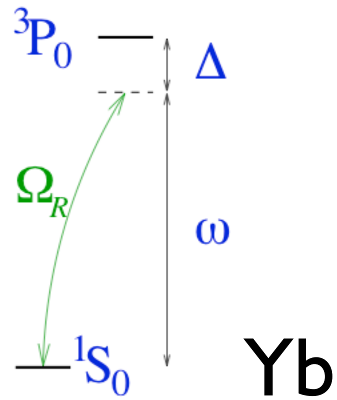

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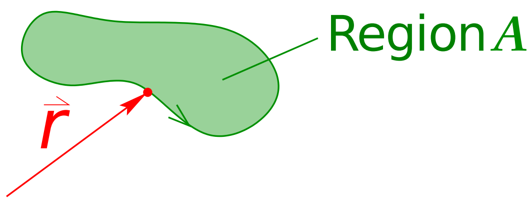
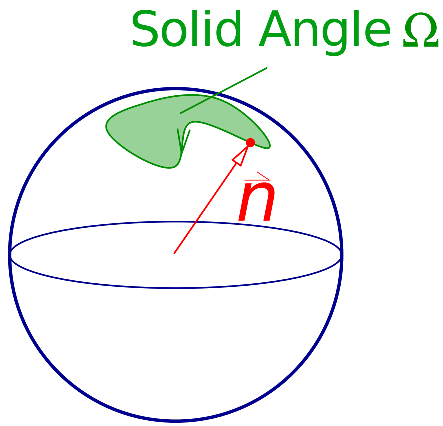
adiabatic motion of atoms *in space* in the optical potential generates a Berry phase

for *N*=2, consider Bloch sphere of unit vector $\vec{n} = \langle\Psi_{\mathbf{r}}|\hat{\vec{\sigma}}|\Psi_{\mathbf{r}}\rangle$

Berry flux generated: $$n_\phi = \frac{1}{8\pi}\epsilon_{ijk}\epsilon_{\mu\nu}n_i\partial_\mu n_j\partial_\nu n_k$$





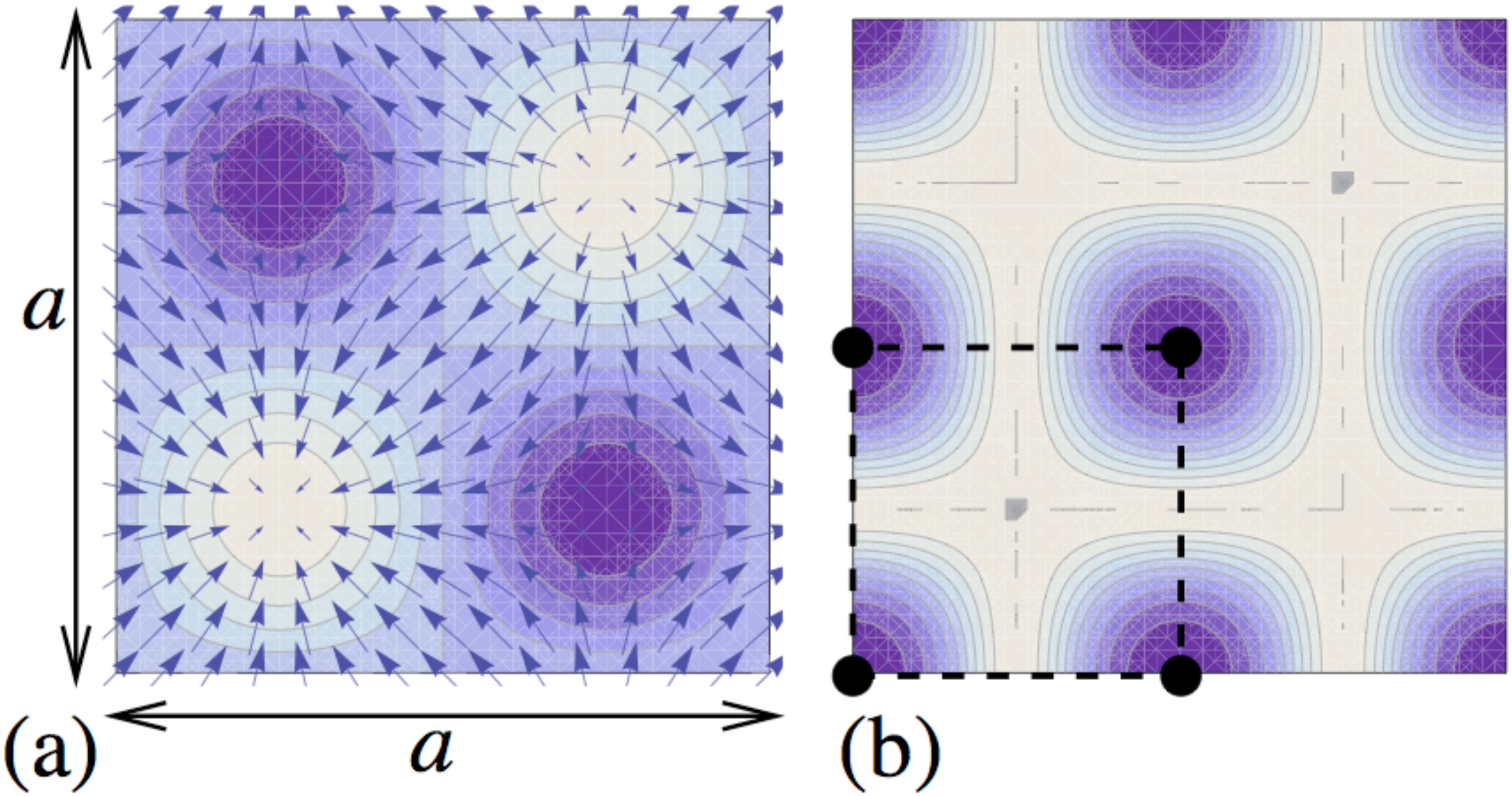
Total flux quanta = # times Bloch vector wraps sphere

$$\int_A n_\phi \, d^2r = \frac{\Omega}{4\pi}$$

Cold Atoms II: Optical Flux Lattices

Nigel Cooper, PRL (2011); N. Cooper & J. Dalibard, EPL (2011)

periodic optical Raman potentials are conveniently located by standing wave Lasers yielding an optical flux lattice of high flux density, here $n_\phi = 1/2a^2$ (fixed)



(a) Local direction of unit Bloch vector $\vec{n}$ (b) Local density of Berry Flux n_ϕ

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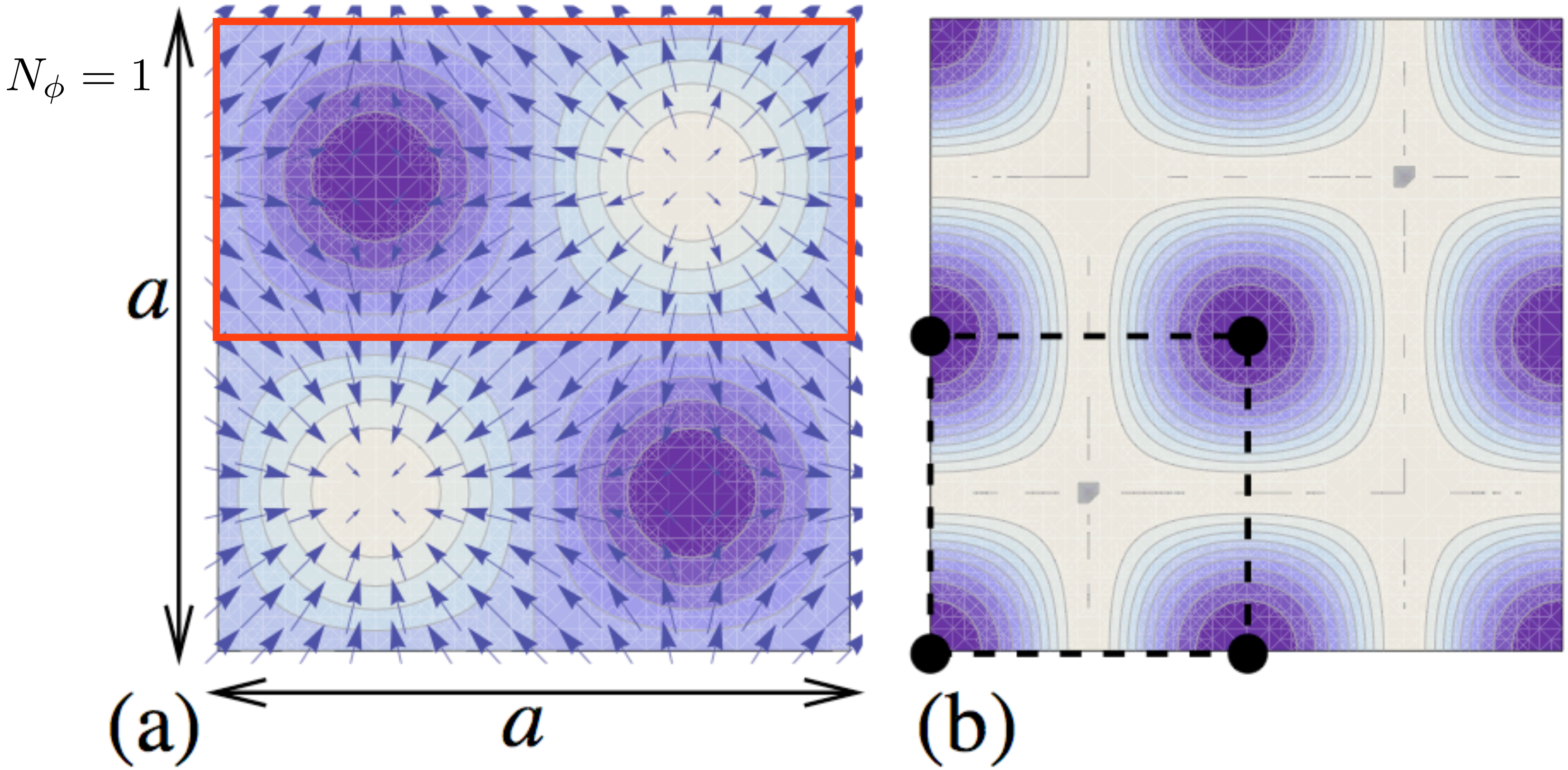

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Fractionalization in Chern bands? Chern #1 Bands = FQHE ?

Proposition: correlated states reproduce the physics of FQHE

Selected for a Viewpoint in *Physics*
PRL **106,** 236804 (2011) PHYSICAL REVIEW LETTERS week ending 10 JUNE 2011

Fractional Quantum Hall States at Zero Magnetic Field

Titus Neupert,[1] Luiz Santos,[2] Claudio Chamon,[3] and Christopher Mudry[1]
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[2]*Department of Physics, Harvard University, 17 Oxford Street, Cambridge, Massachusetts 02138, USA*
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Nearly Flatbands with Nontrivial Topology

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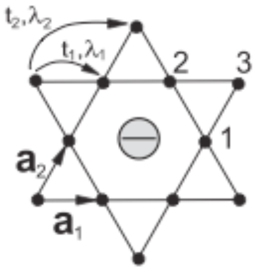

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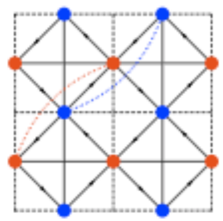

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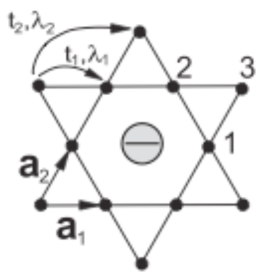

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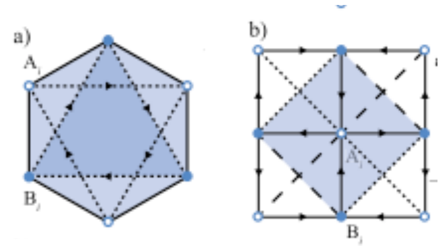

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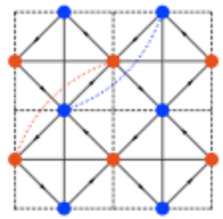

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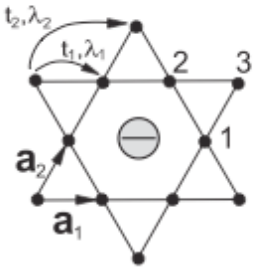

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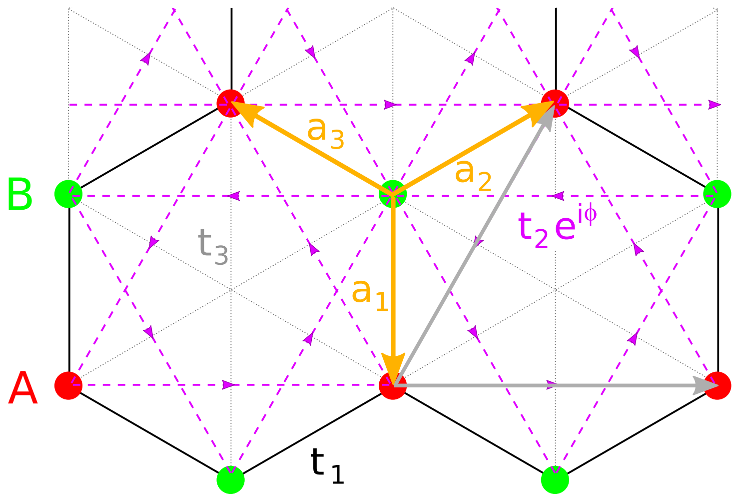
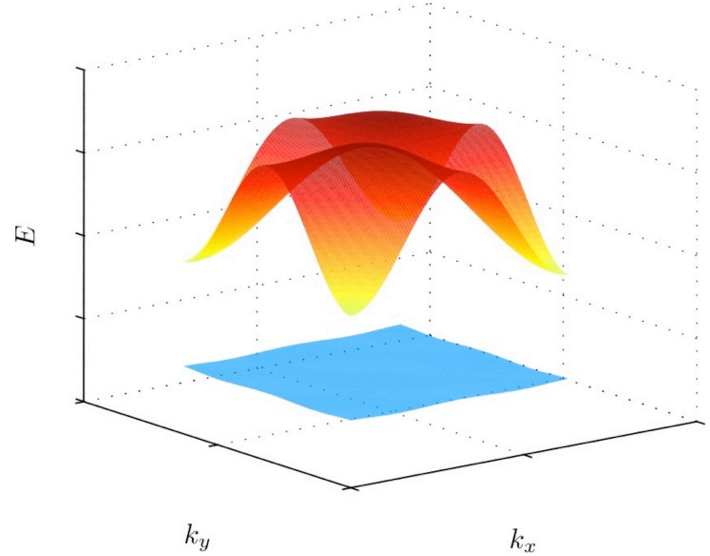
Characteristic example: The Haldane Model

F.D.M. Haldane, PRL (1988), Neupert *et al.* PRL (2011)





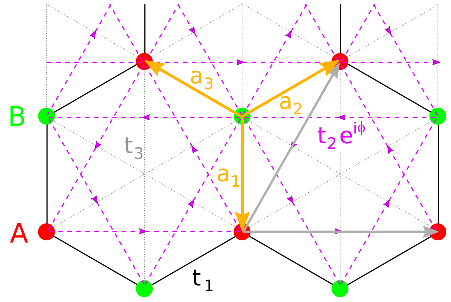
$$\mathcal{H} = -\,t_1 \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} \left(\hat{a}^\dagger_{\mathbf{r}} \hat{a}_{\mathbf{r}} + h.c. \right) - t_2 \sum_{\langle\langle \mathbf{r}\mathbf{r}' \rangle\rangle} \left(\hat{a}^\dagger_{\mathbf{r}} \hat{a}_{\mathbf{r}} e^{i\phi_{\mathbf{r}\mathbf{r}'}} + h.c. \right)$$

$$-\,t_3 \sum_{\langle\langle\langle \mathbf{r}\mathbf{r}' \rangle\rangle\rangle} \left(\hat{a}^\dagger_{\mathbf{r}} \hat{a}_{\mathbf{r}} + h.c. \right) + \frac{U}{2} \sum_{\mathbf{r}} \hat{n}_{\mathbf{r}} (\hat{n}_{\mathbf{r}} - 1)$$

- *tight binding* model in *real space* on hexagonal lattice
- with fine-tuned hopping parameters: obtain flat lower band, e.g. values [D. Sheng, PRL (2011)]

$$t_1 = 1,\ t_2 = 0.60,\ t_2 = -0.58 \text{ and } \phi = 0.4\pi$$

Topological (flat) bands in two-dimensions

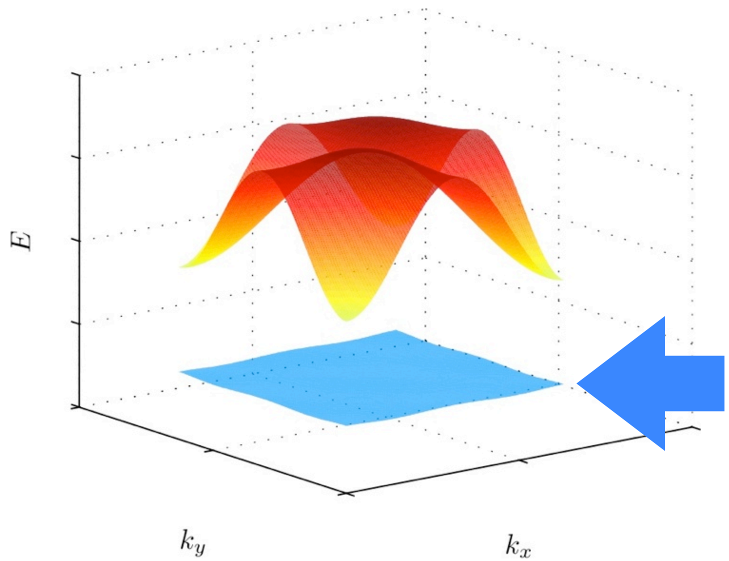


- diagonalize Hamiltonian by Fourier transform

$$\mathcal{H} = \sum_{\mathbf{k}} \hat{a}^\dagger_{\mathbf{k},\alpha} h_{\alpha\beta}(\mathbf{k}) \hat{a}_{\mathbf{k},\beta}$$

Bloch states

$$h_{\alpha\beta}(\mathbf{k}) u^n_\beta(\mathbf{k}) = \epsilon_n(\mathbf{k}) u^n_\alpha(\mathbf{k})$$



- study Berry curvature in n^{th} band:

Berry connection: $\mathcal{A}(n, \mathbf{k}) = -i \sum_\alpha u^{n*}_\alpha(\mathbf{k}) \nabla_{\mathbf{k}} u^n_\alpha(\mathbf{k})$

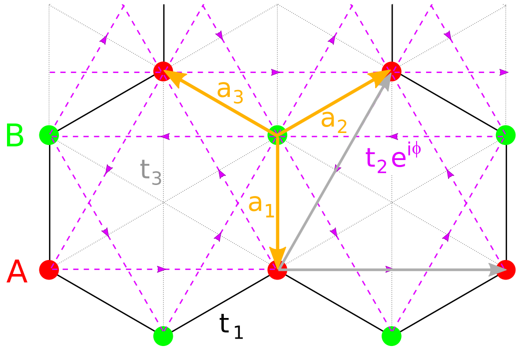
Berry curvature: $\mathcal{B}(k) = \nabla_{\mathbf{k}} \wedge \boldsymbol{\mathcal{A}}(k)$

Chern number: $C = \frac{1}{2\pi} \int_{BZ} d^2\mathbf{k}\, \mathcal{B}(\mathbf{k})$

Flux Lattices vs Chern Bands

CBs = tight binding model in *real* space with topological flat bands

real space



reciprocal space

real space *reciprocal* space

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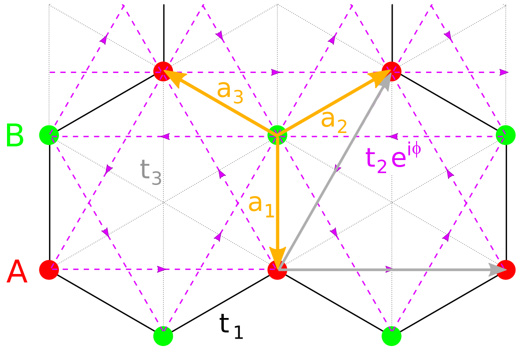
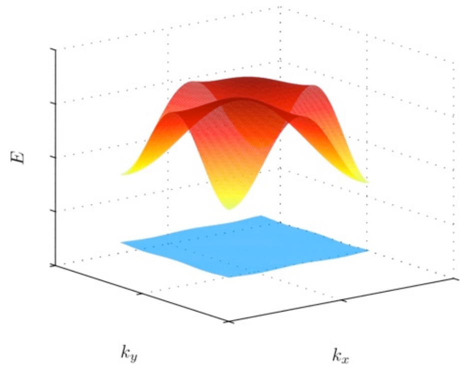
real space



reciprocal space



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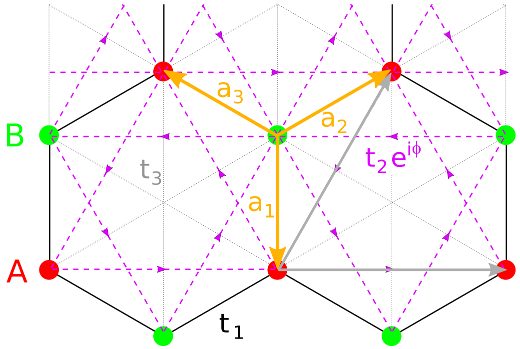
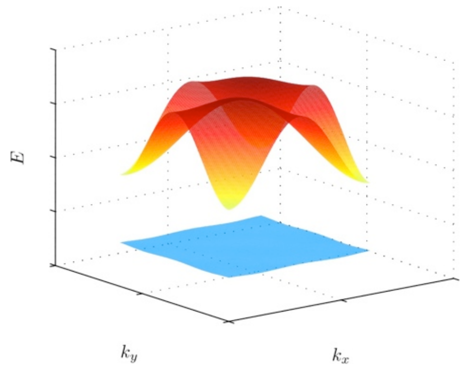
real space

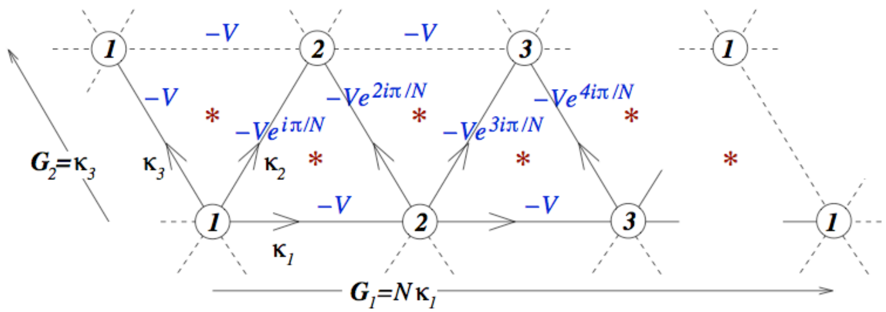
reciprocal space





Flux Lattice = tight binding model in *reciprocal* space with topological flat bands

N. R. Cooper, R. Moessner, PRL (2012) [arxiv:1208.4579]



absorption of photons = momentum + state transfers

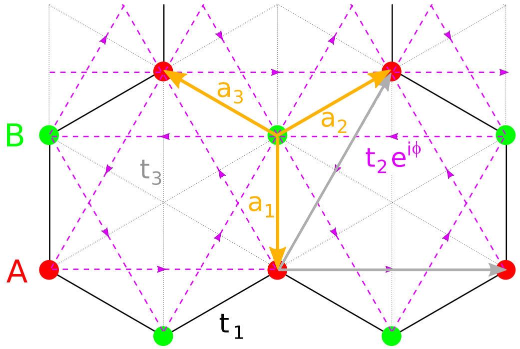
real space

reciprocal space

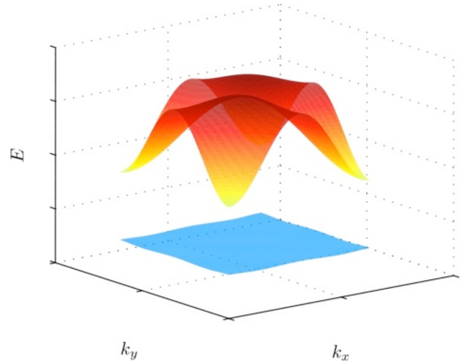
Flux Lattices vs Chern Bands

CBs = tight binding model in *real* space with topological flat bands

real space



reciprocal space



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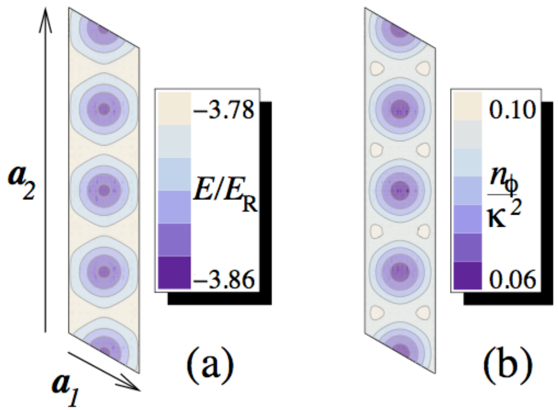
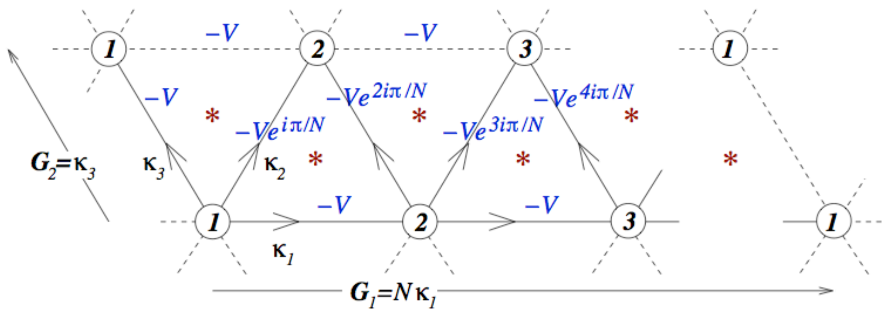




absorption of photons = momentum + state transfers

real space

reciprocal space

Overview of underlying band structures

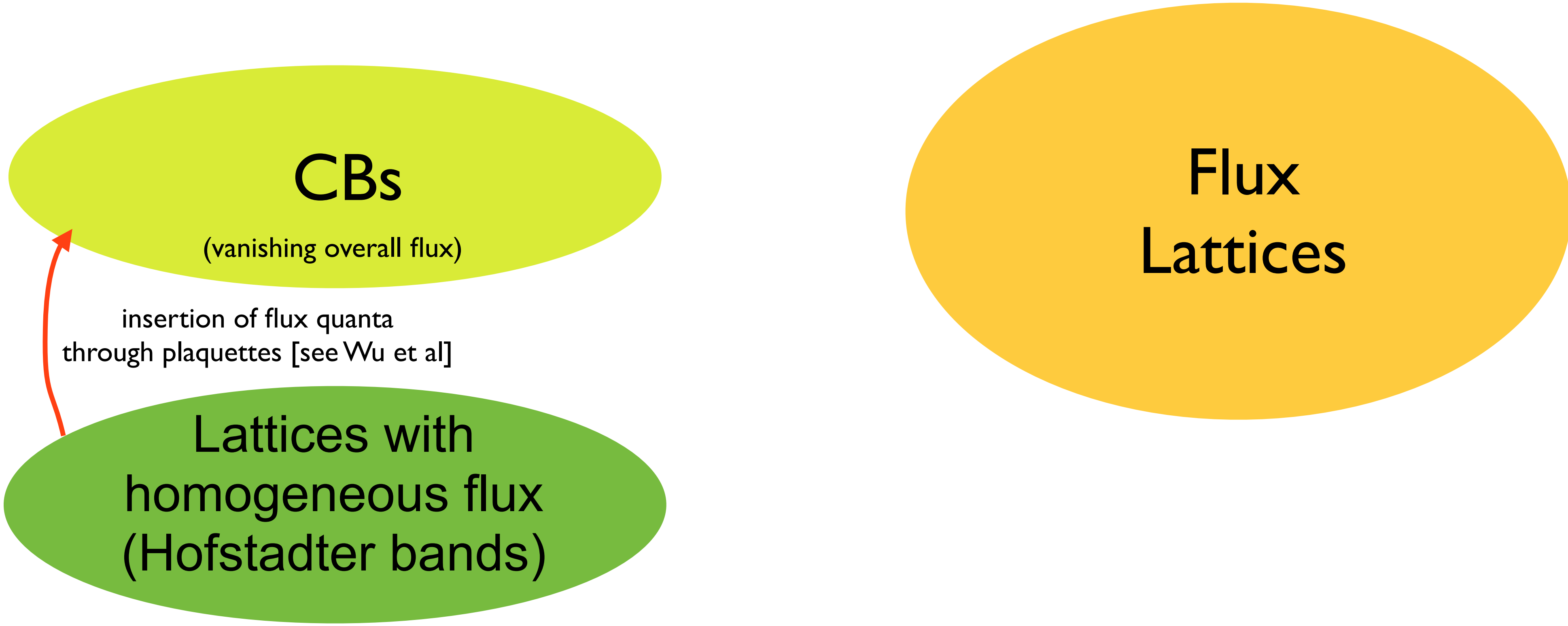
CBs

(vanishing overall flux)

Flux
Lattices

Lattices with
homogeneous flux
(Hofstadter bands)

Overview of underlying band structures



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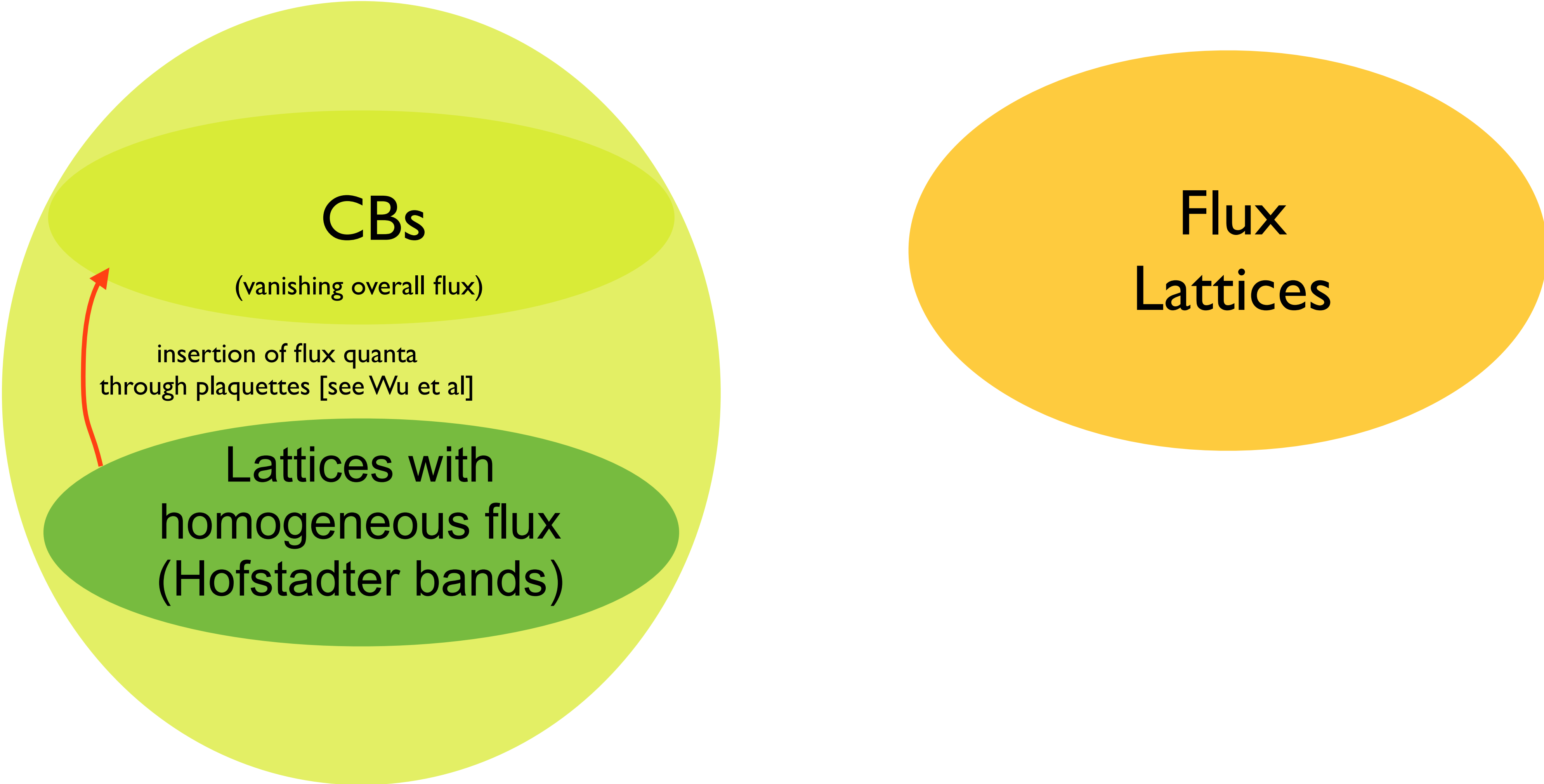

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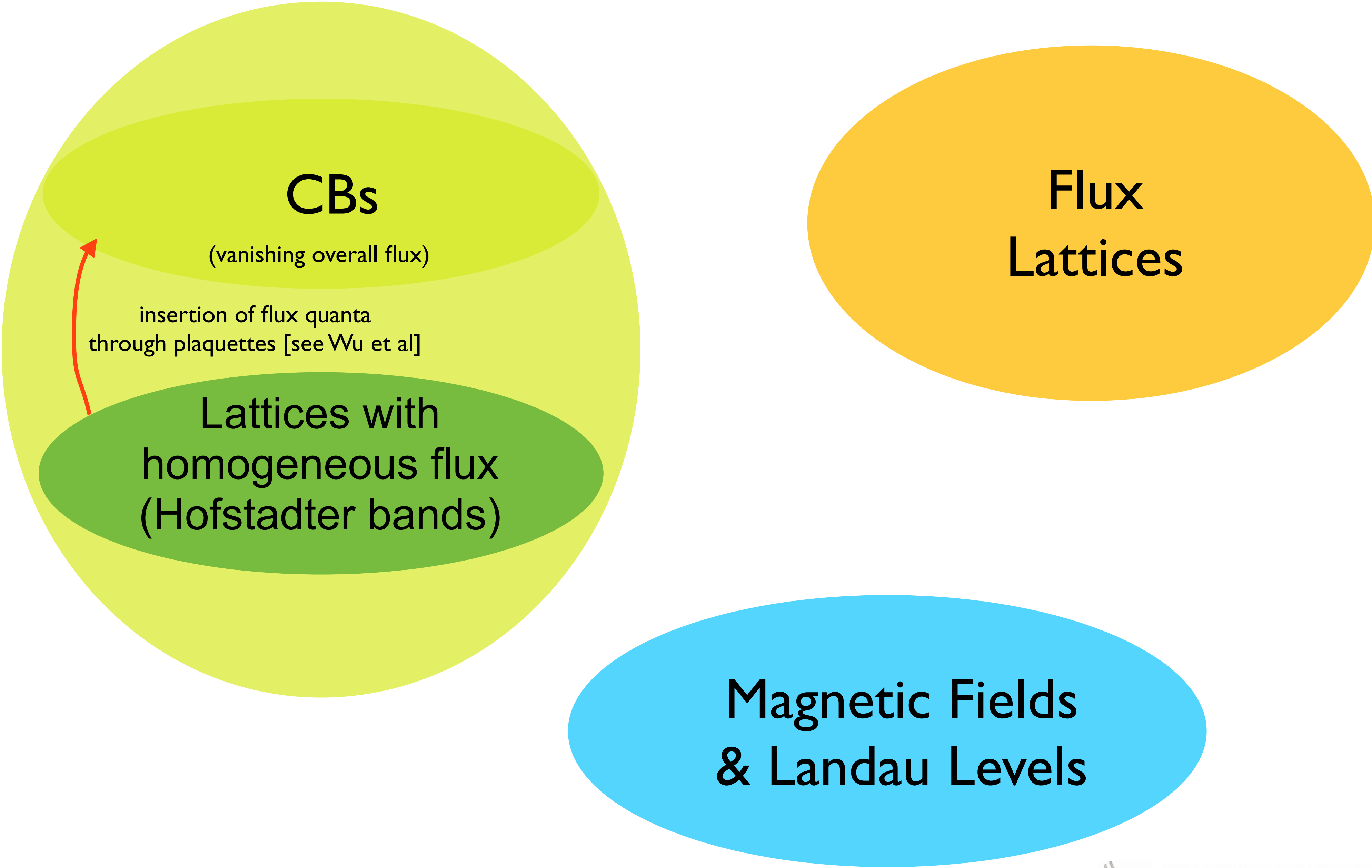

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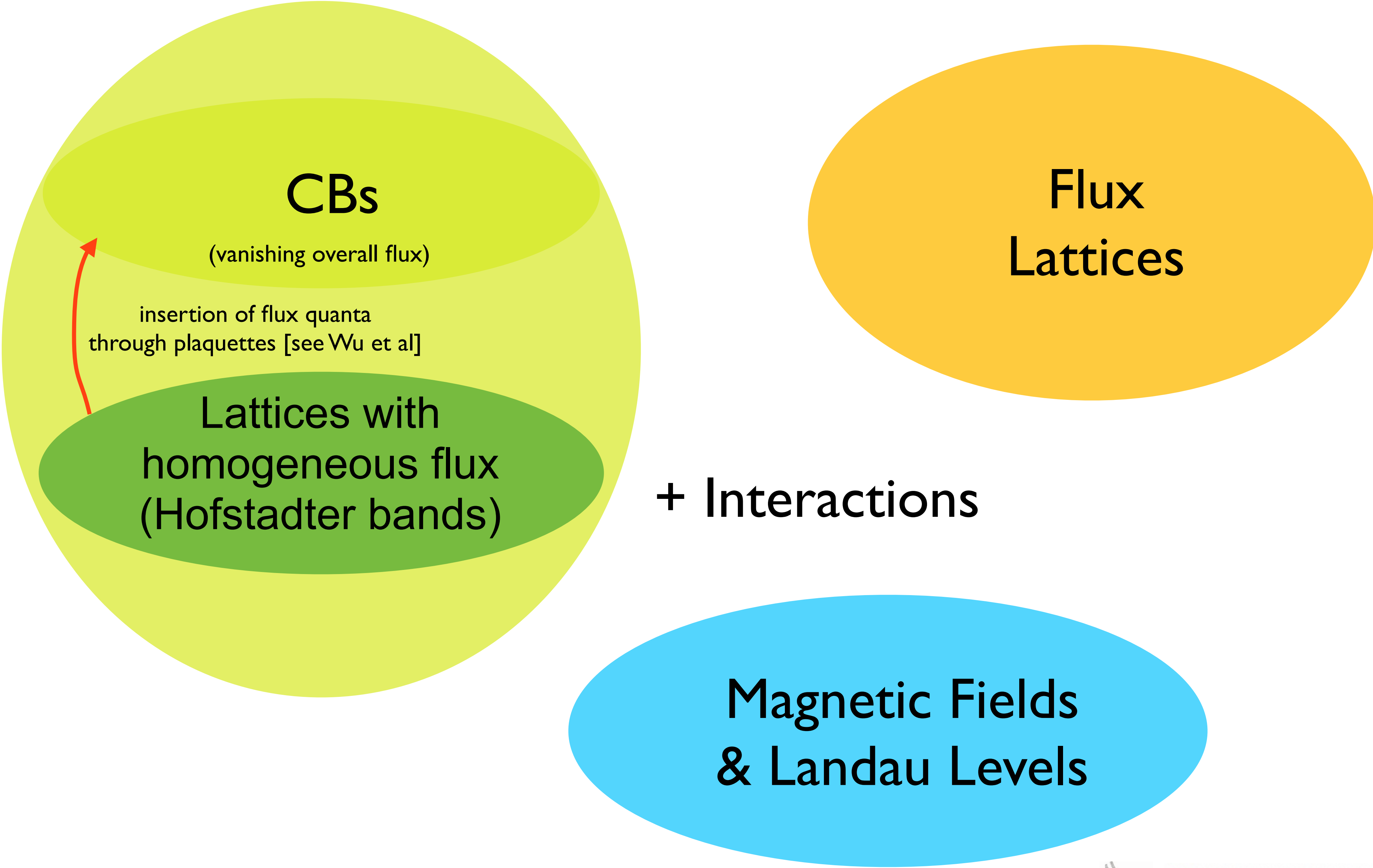
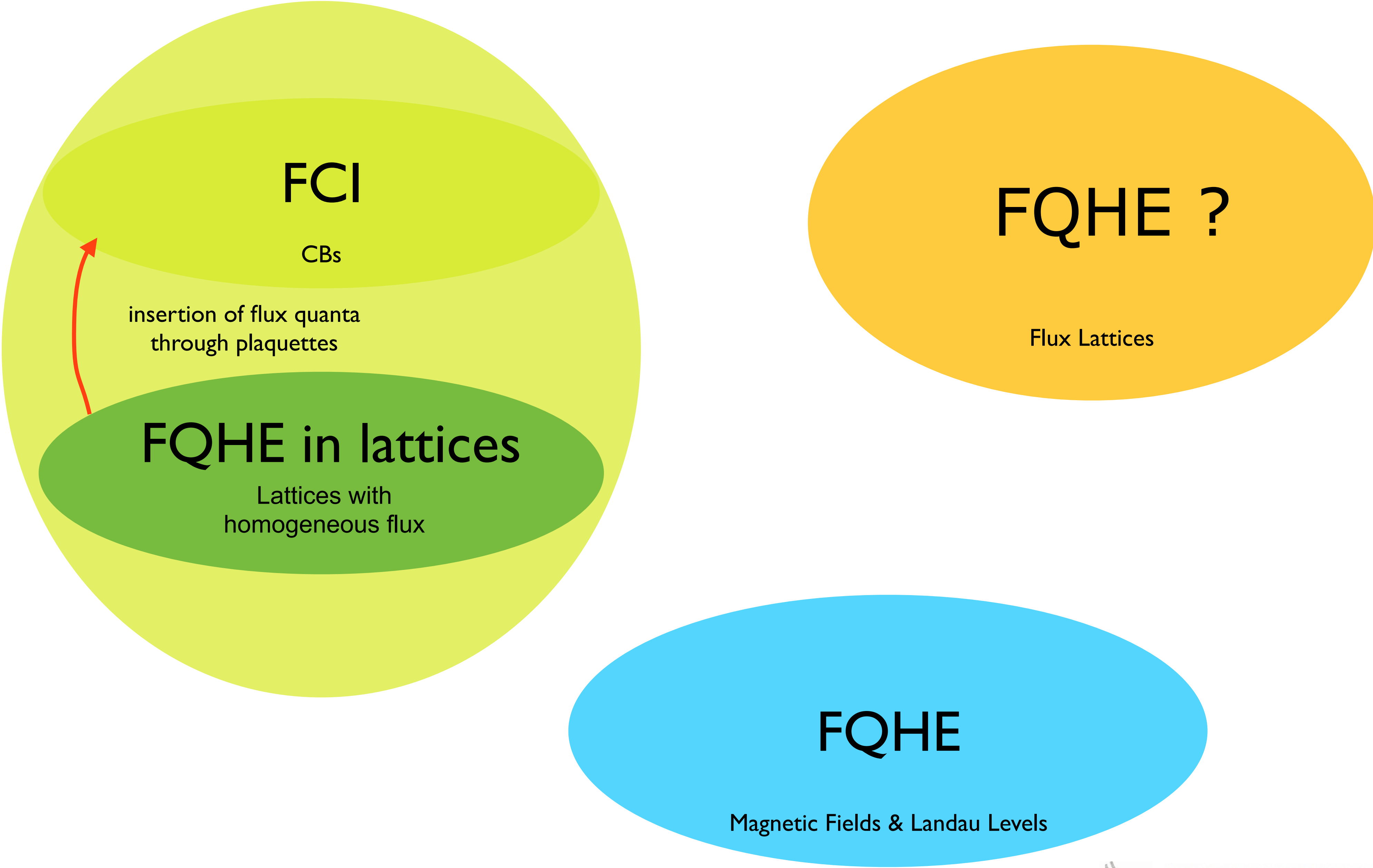


Interaction driven phases in these bands



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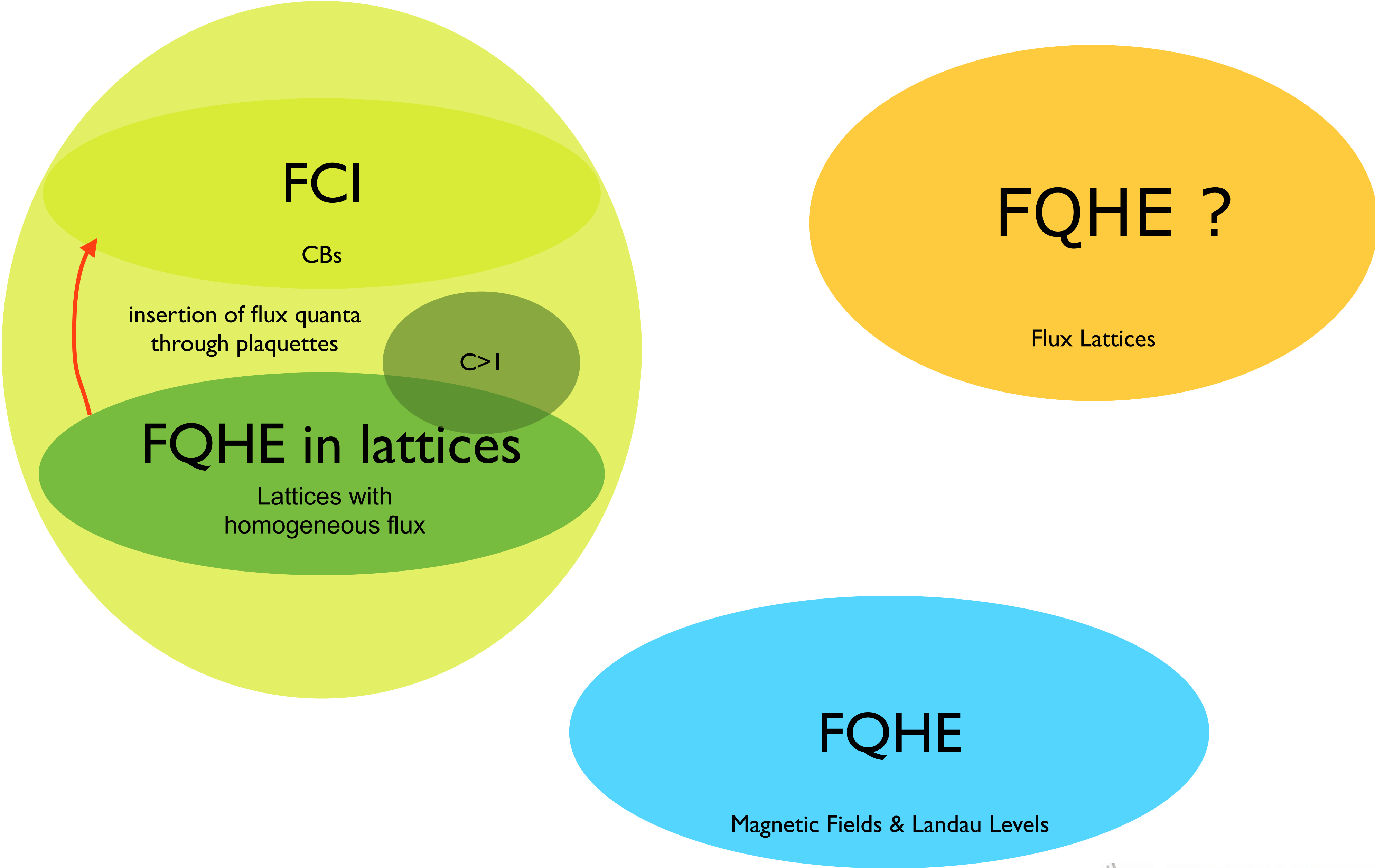

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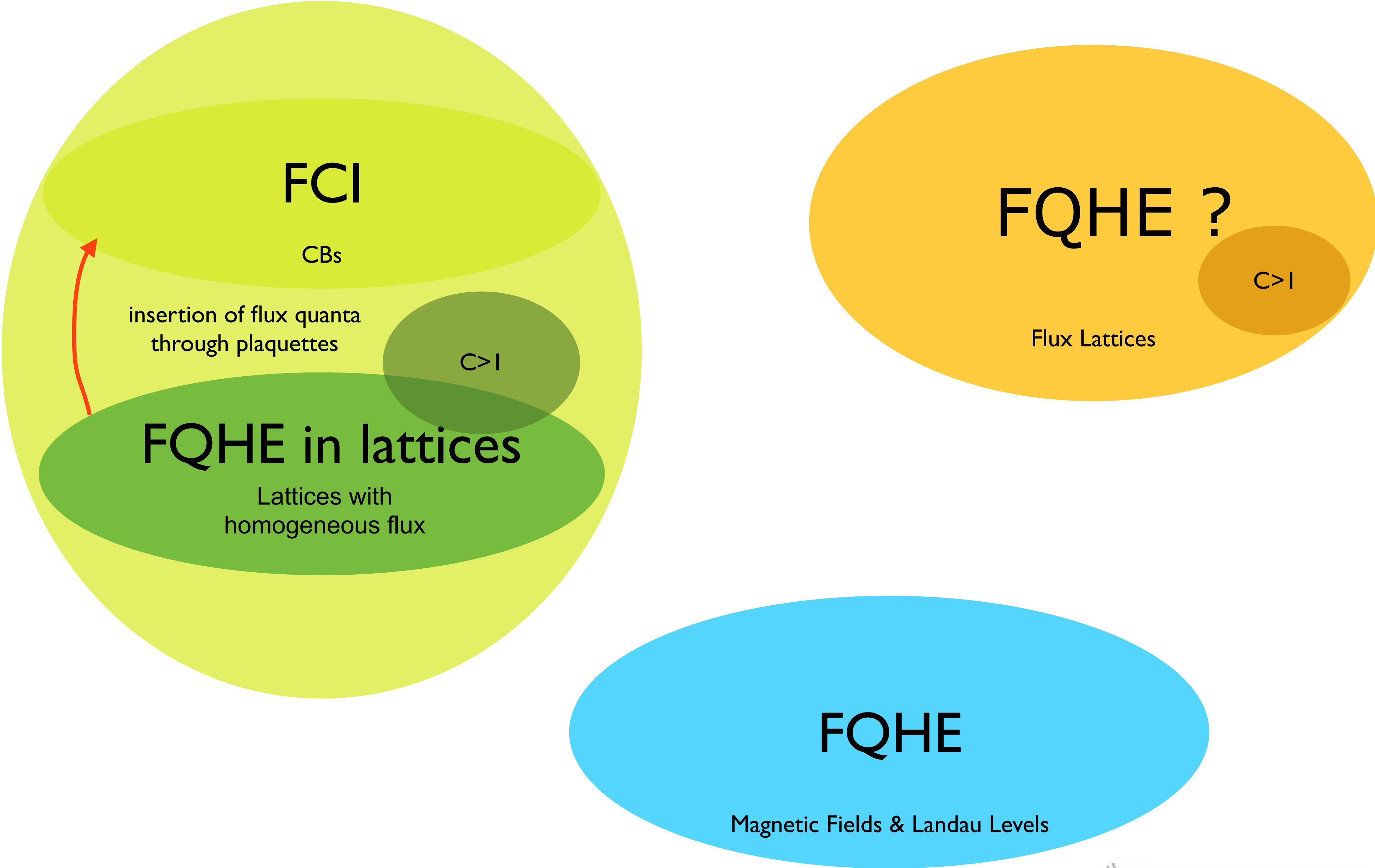
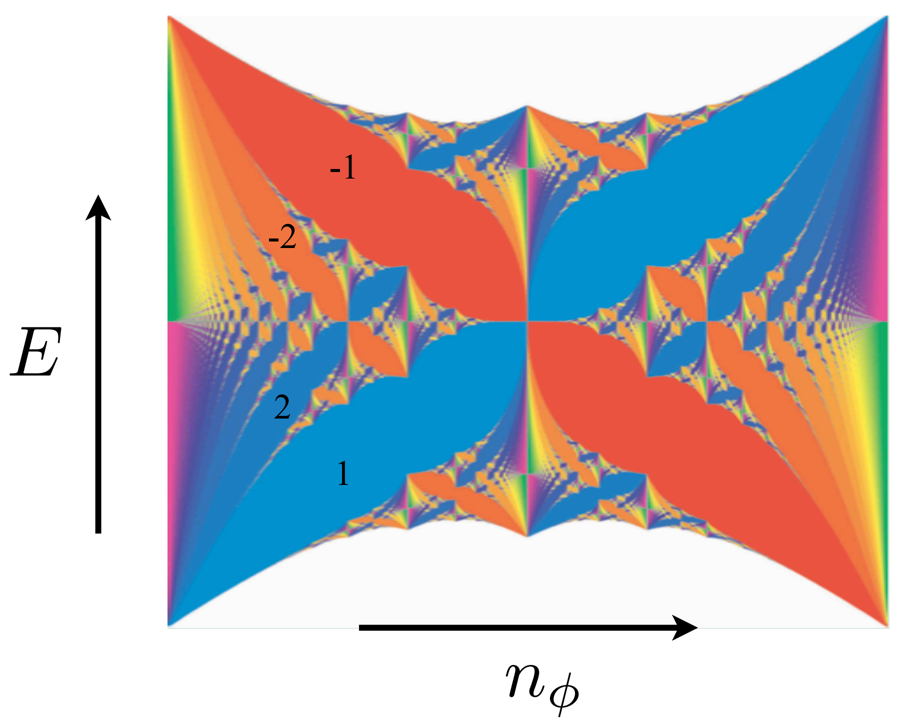


Strongly correlated states from the Hofstadter spectrum

- Hofstadter spectrum provides bands of all Chern numbers [Avron et al.]



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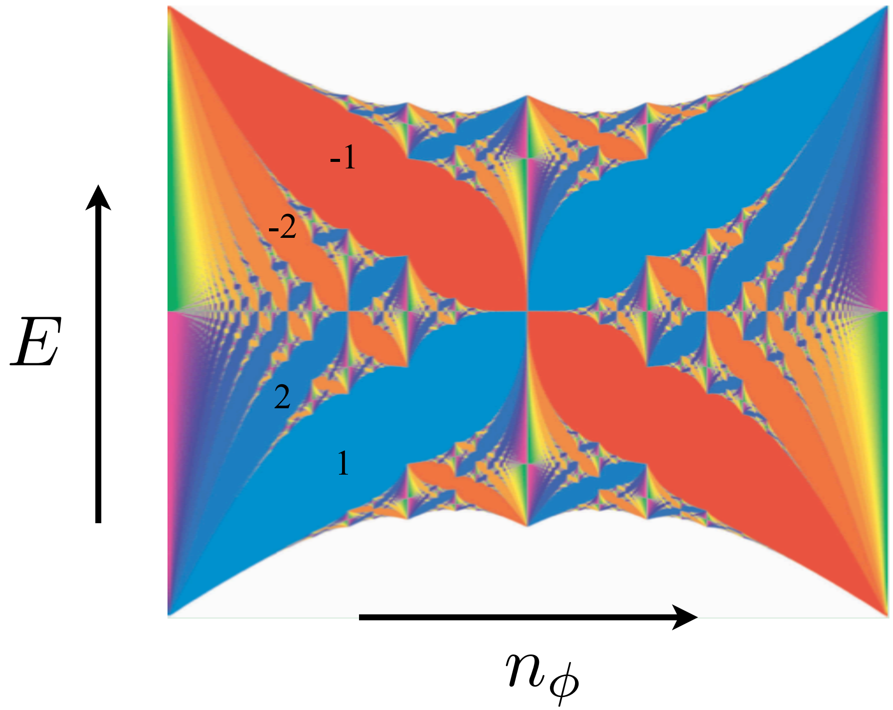
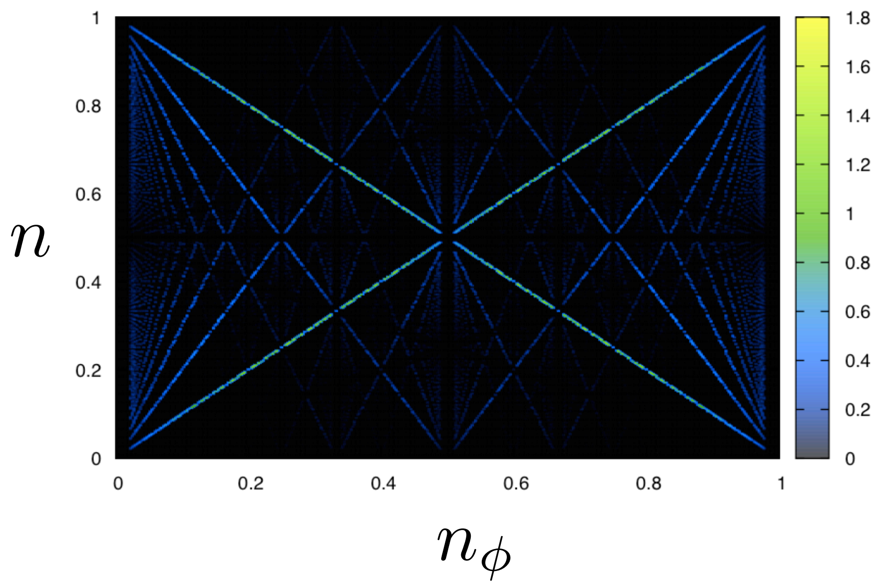


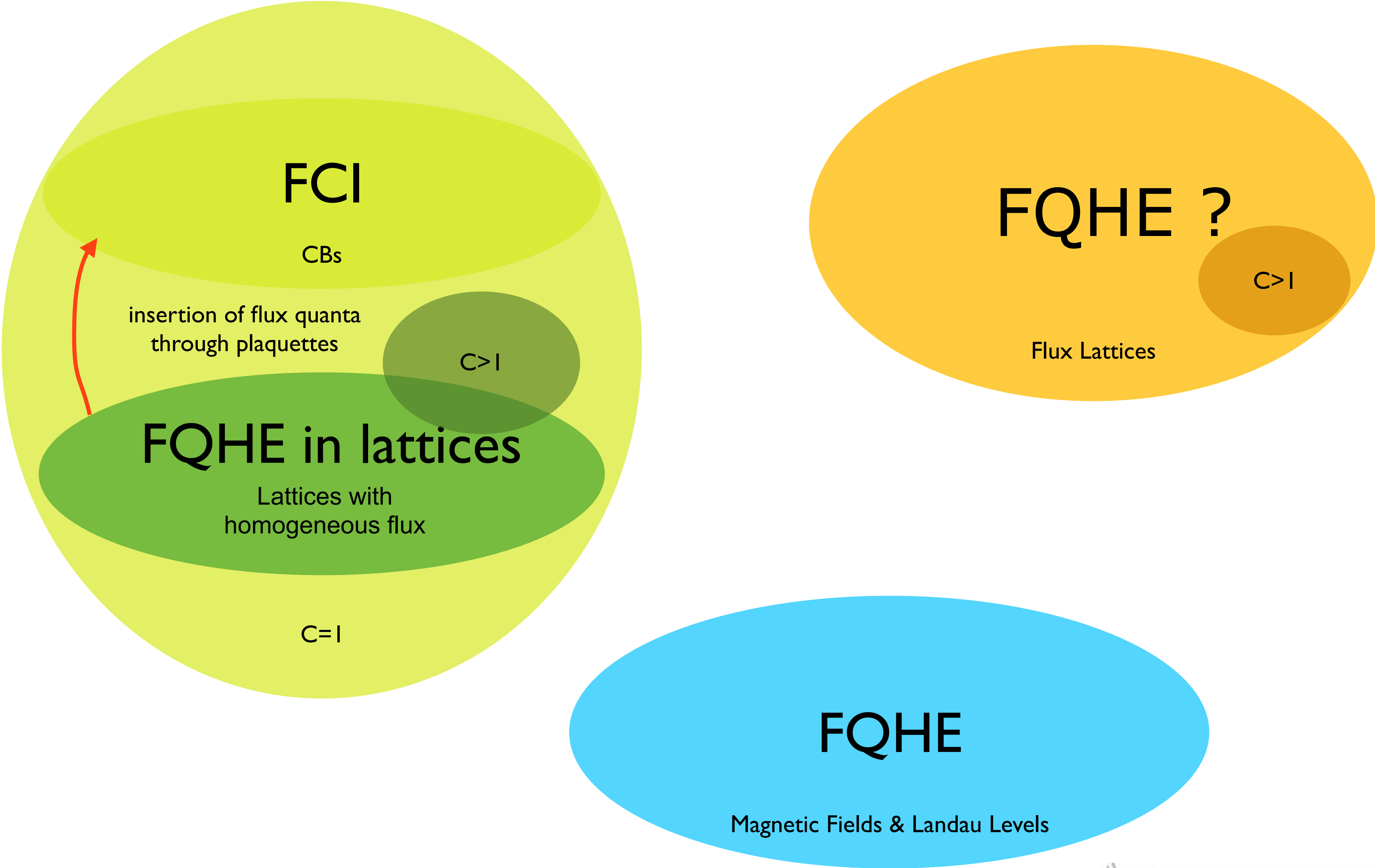
- Interactions stabilize fractional quantum Hall liquids in these bands!
- CF Theory: GM & N. R. Cooper, PRL (2009)



- Near rational flux density: LL's with additional pseudospin index

R. Palmer & D. Jaksch PRL 2006
L. Hormozi et al, PRL 2012

Interrelations between different interacting problems



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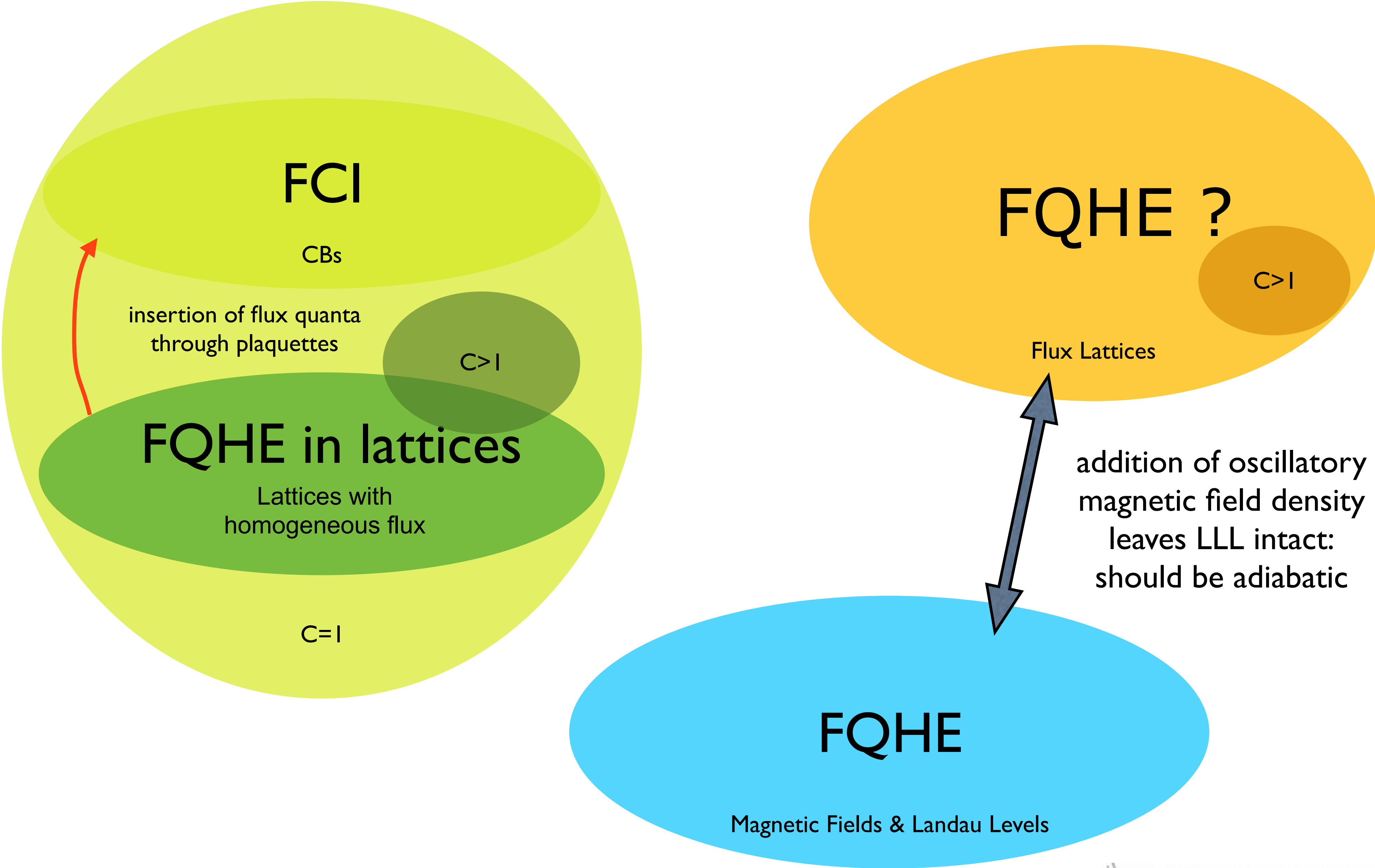

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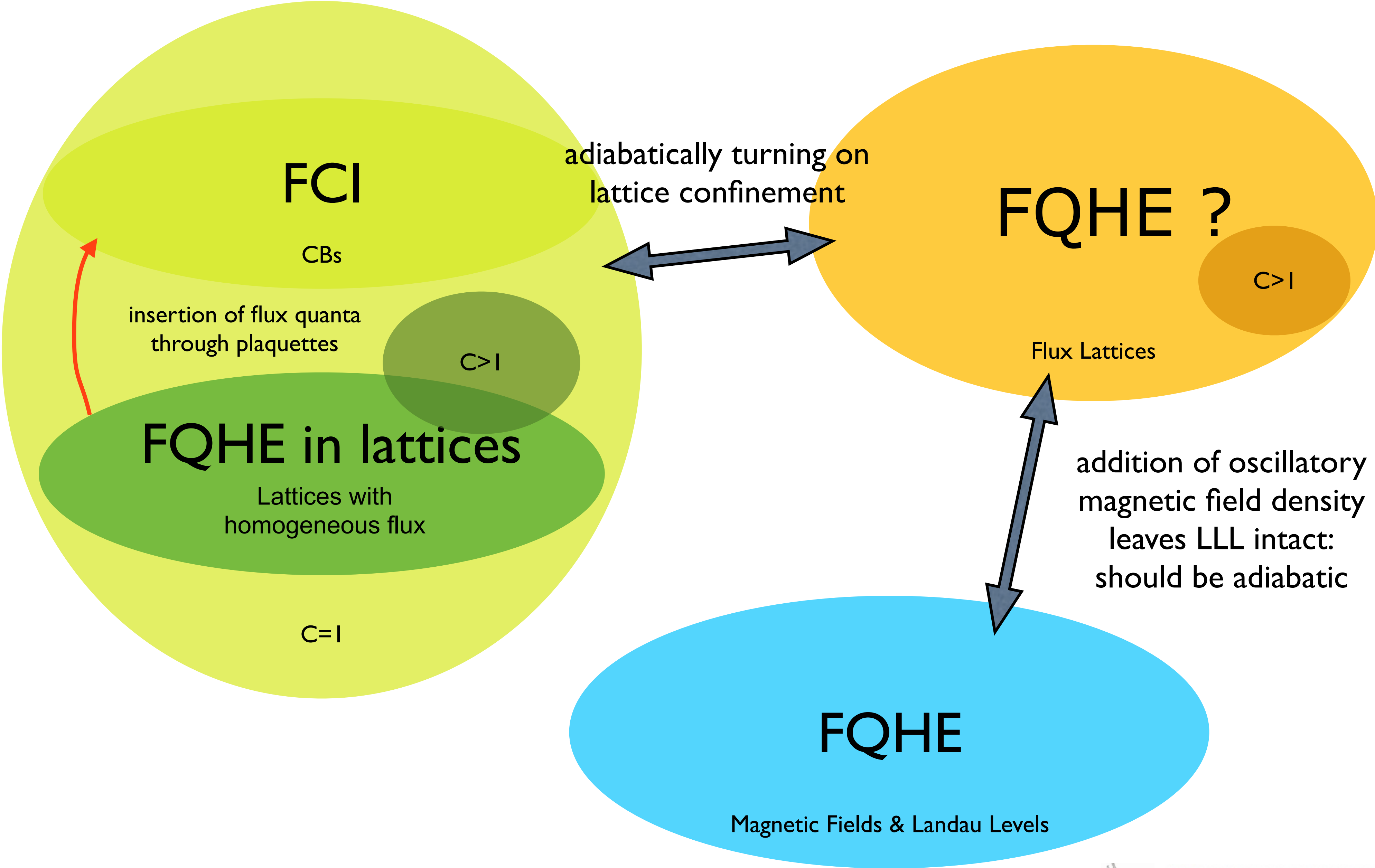

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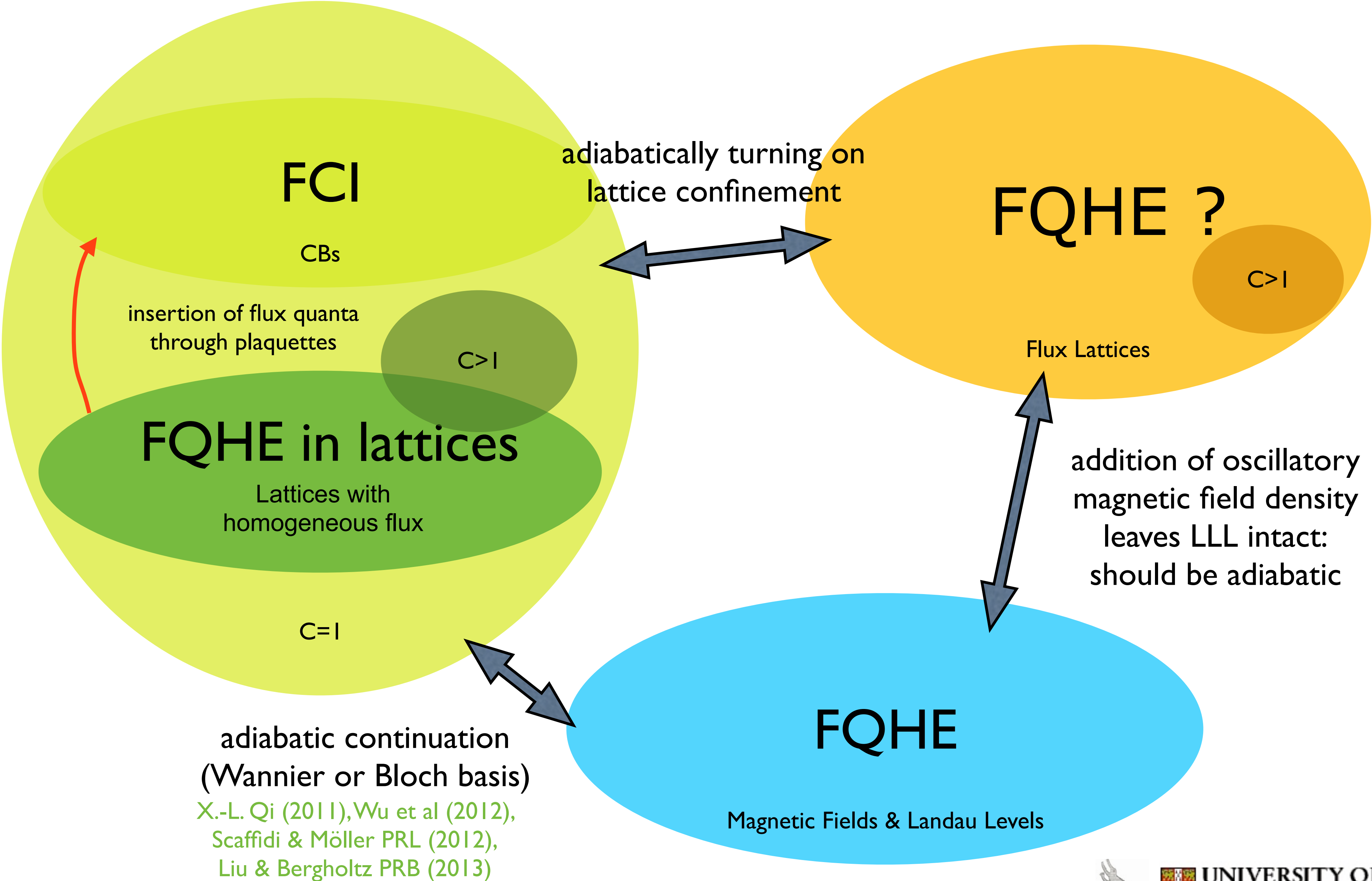
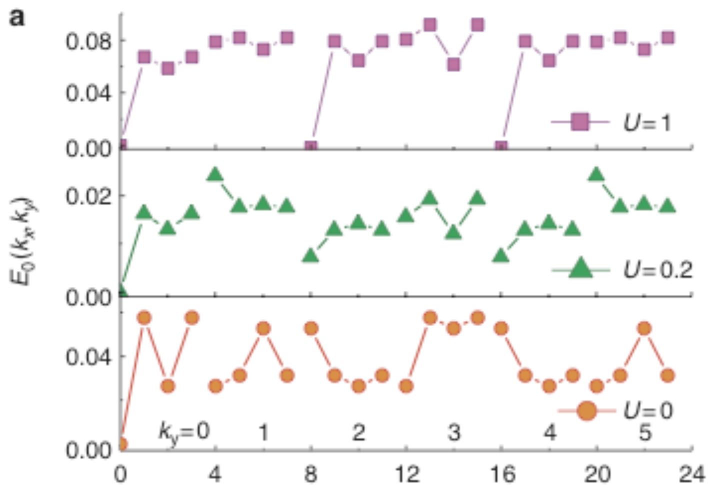


Numerical evidence for “Fractional Chern Insulators”

- existence of a gap & groundstate degeneracy [checkerboard lattice]
- chern number of groundstate manifold



[D. Sheng]

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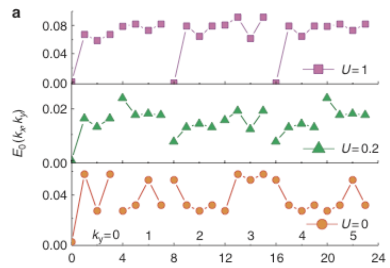
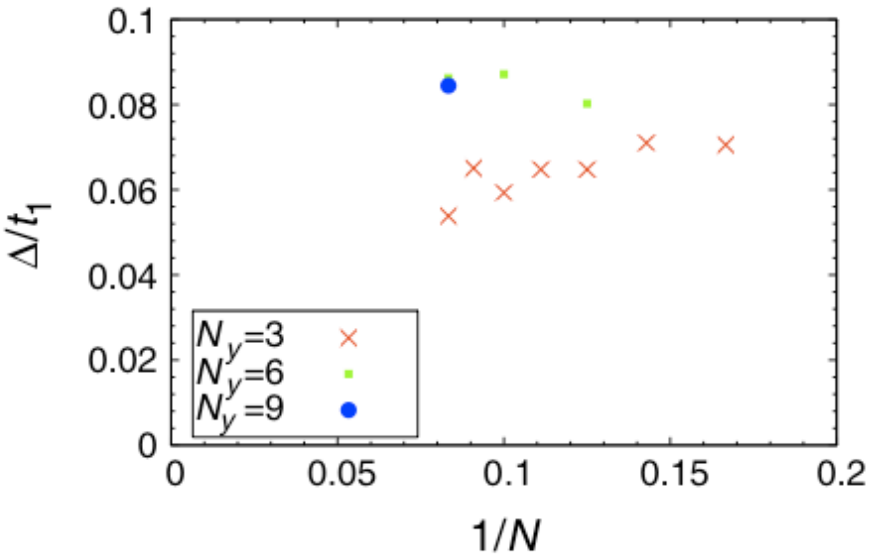
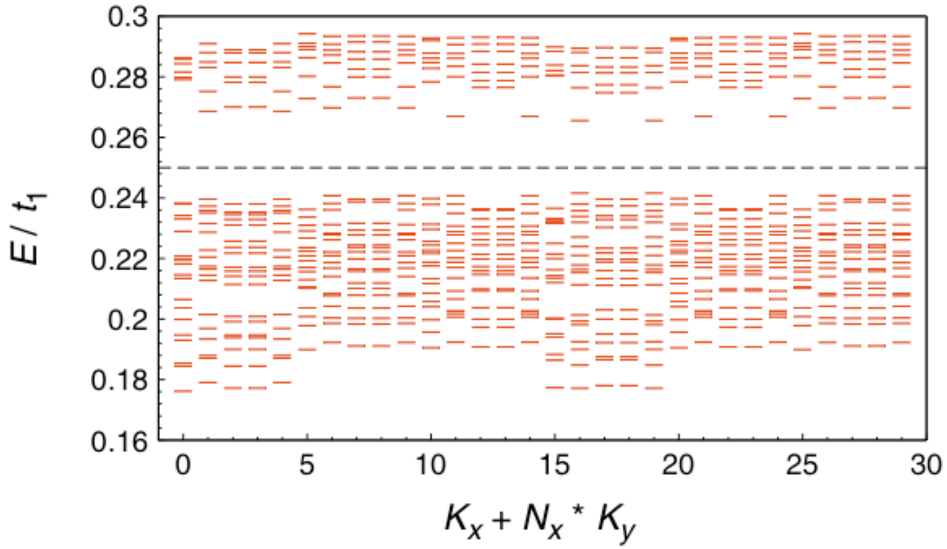


- Finite size scaling of gap



- Particle Entanglement Spectra : count of excitations matches FQHE (here - Laughlin state)



"Fractional Chern Insulators (FCI)" [N. Regnault & A. Bernevig, PRX '11]

Numerical evidence for “Fractional Chern Insulators”

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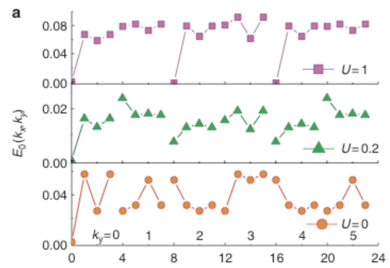

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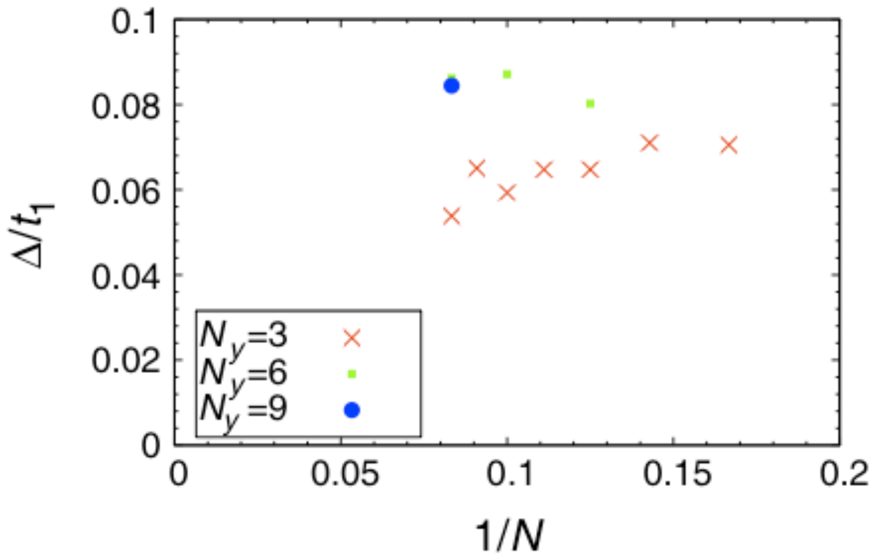

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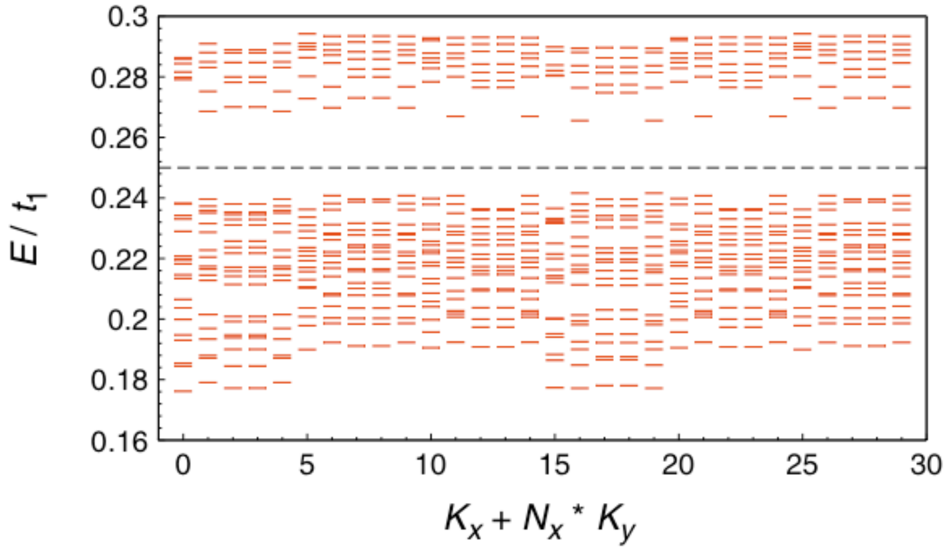

“Fractional Chern Insulators (FCI)” [N. Regnault & A. Bernevig, PRX ’11]

- Strong numerical evidence for QHE physics, but no clear organising principle for different lattice models

Understanding Fractional Quantum Hall states

- Single particle states are analytic functions in symmetric gauge

$$\vec{A} = \frac{1}{2}\vec{r} \wedge \vec{B}$$

$$\phi_m \propto z^m e^{-|z|^2/4\ell_0}$$

$$z_j = x_j + iy_j$$

- Many particle states are still analytic functions - can write explicitly!

e.g. Laughlin:

$$\Psi_{\nu=\frac{1}{m}} = \prod_{i<j}(z_i - z_j)^m e^{-\sum_i |z_i|^2/4\ell_0}$$

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✦ Incompressibility ✦ Quasiparticle excitations: charge / statistics

✦ Correlations / You name the observable...

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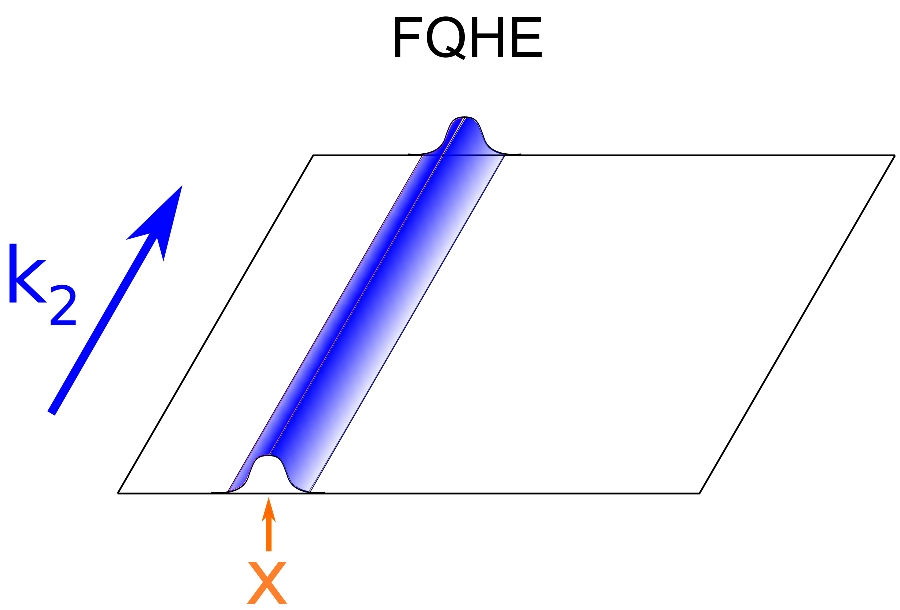
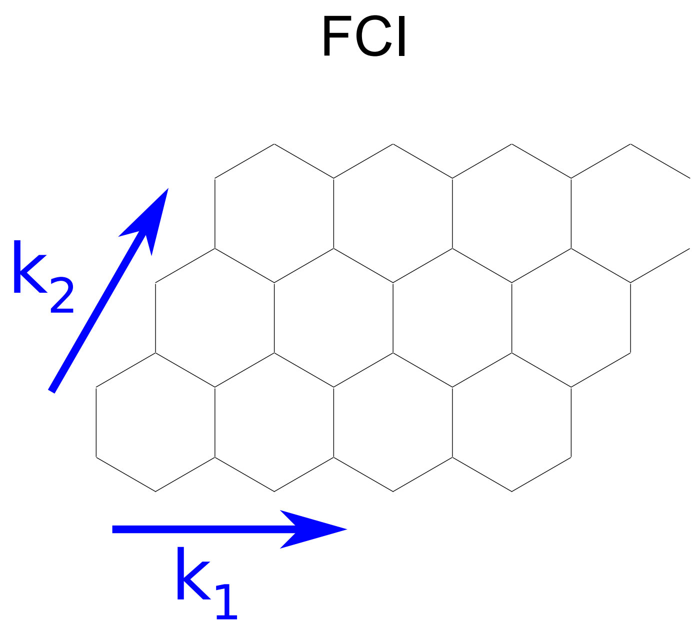
Can we construct wavefunctions for Chern Insulators?

Mapping from FQHE to FCI: Single Particle Orbitals





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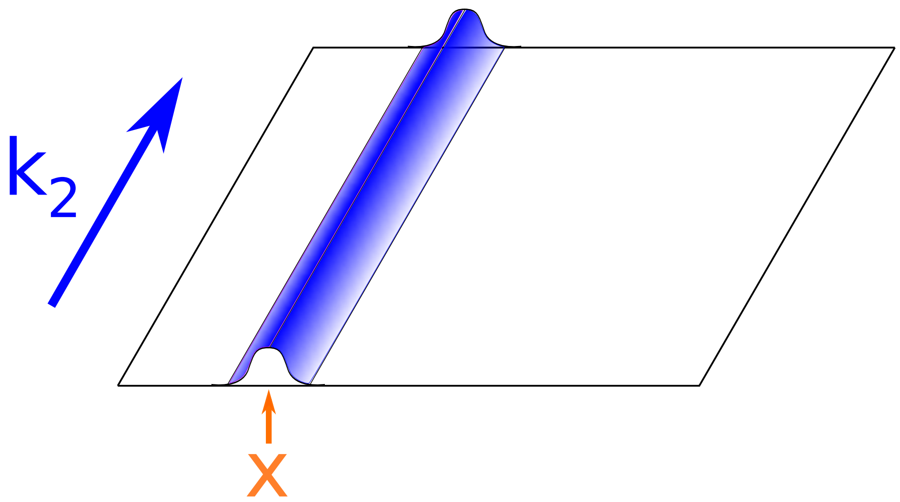
Mapping from FQHE to FCI: Single Particle Orbitals

FQHE



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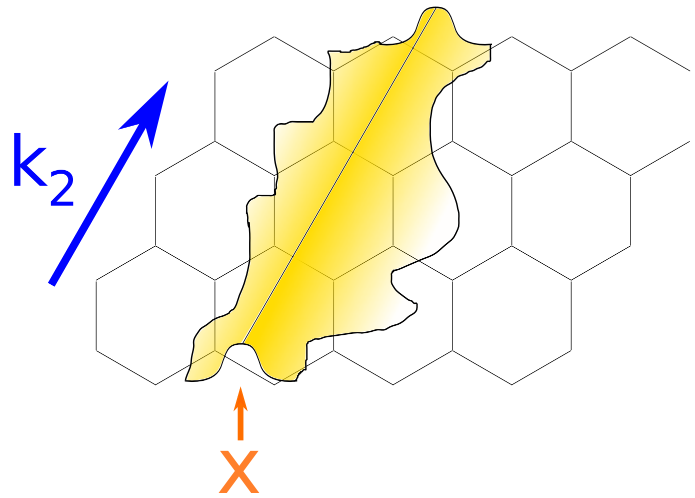

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- keep translational invariance in y (cannot create fully localized Wannier state if C>0!)

$$|W(x,k_y)\rangle = \sum_{k_x} f_{k_x}^{(x,k_y)} |k_x,k_y\rangle$$

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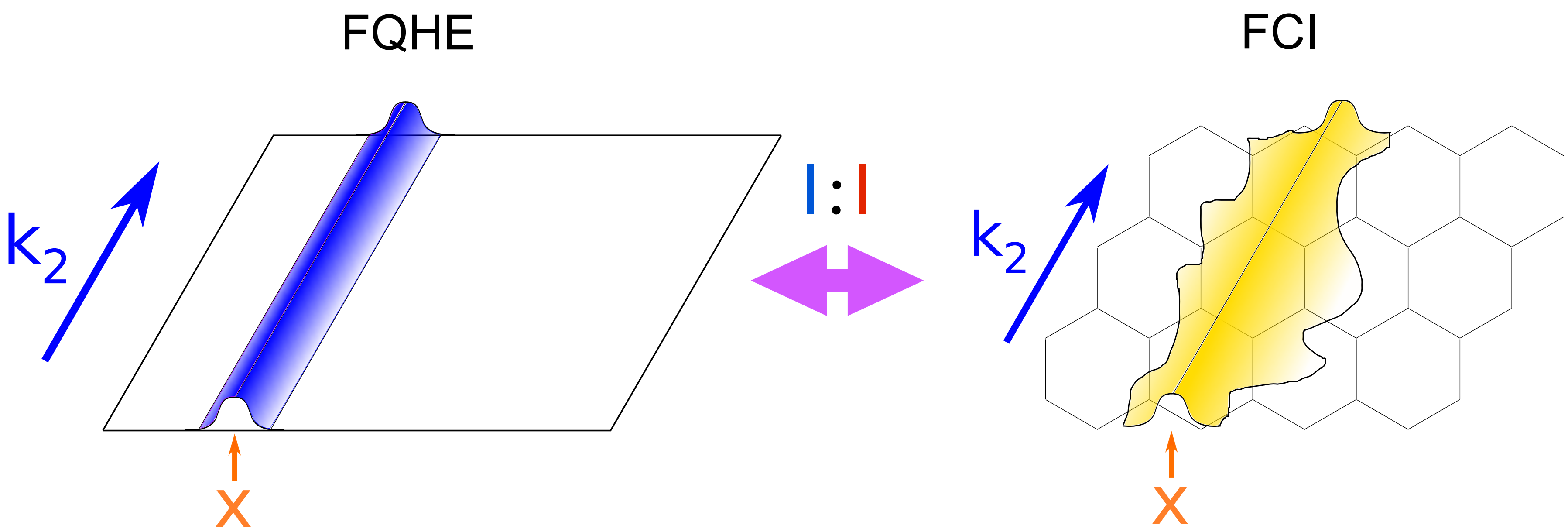

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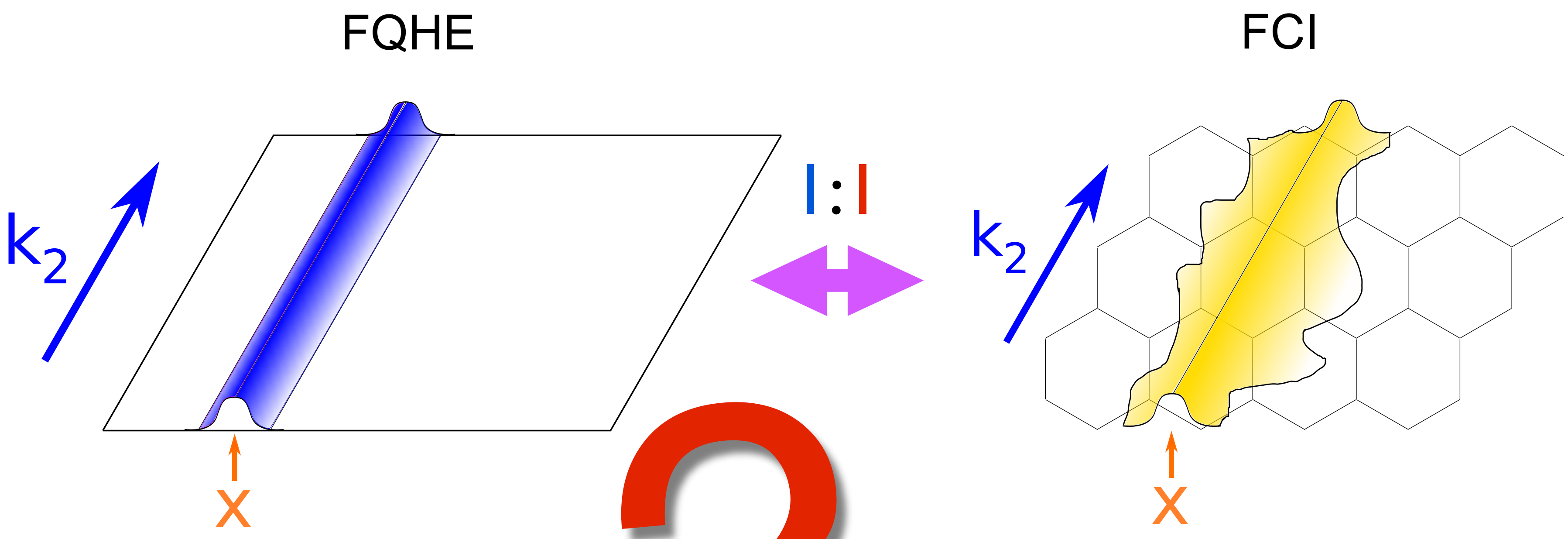

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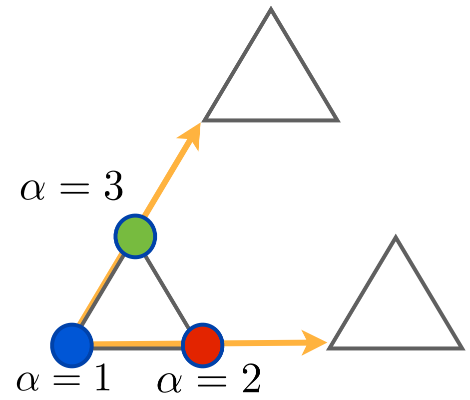
Wannier states in Chern bands

- Some formalism

$$\mathcal{H} = \sum_{\mathbf{k}} \hat{a}^{\dagger}_{\mathbf{k},\alpha} h_{\alpha\beta}(\mathbf{k}) \hat{a}_{\mathbf{k},\beta}$$ Hamiltonian

$$h_{\alpha\beta}(\mathbf{k}) u^n_\beta(\mathbf{k}) = \epsilon_n(\mathbf{k}) u^n_\alpha(\mathbf{k})$$ Eigenstates

$$\mathcal{A}(n, \mathbf{k}) = -i \sum_\alpha u^{n*}_\alpha(\mathbf{k}) \nabla_{\mathbf{k}} u^n_\alpha(\mathbf{k})$$ Berry connection



$$c^\dagger_{\mathbf{k},\alpha} = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}} e^{i\mathbf{k}\cdot(\mathbf{R}+\boldsymbol{\delta}_\alpha)} c^\dagger_{\mathbf{R},\alpha}$$

- construction of a Wannier state at fixed k_y

$$|W(x, k_y)\rangle = \frac{\chi(k_y)}{\sqrt{L_x}} \sum_{k_x}$$

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Fourier transform

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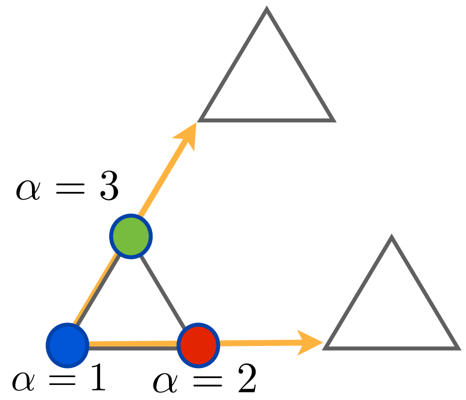

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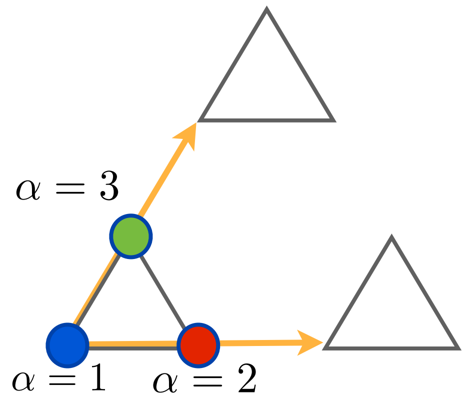
'Parallel transport' of phase

Fourier transform

Berry connection indicates change of phase due to displacement in BZ

Wannier states in Chern bands



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'Parallel transport' of phase — 'Polarization' — Fourier transform

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ensures periodicity of WF in $k_y \to k_y + 2\pi$

Wannier states in Chern bands

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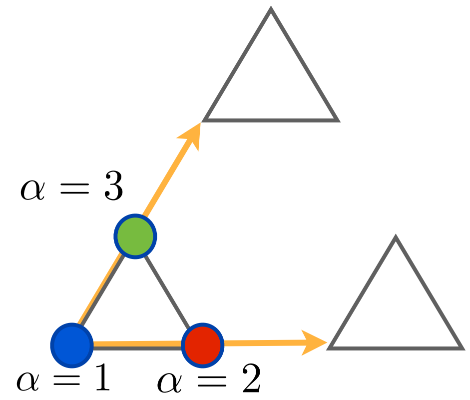

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ky-dependent phase factor, or `gauge'

'Parallel transport' of phase

Berry connection indicates change of phase due to displacement in BZ

'Polarization'

ensures periodicity of WF in $k_y \rightarrow k_y + 2\pi$

Fourier transform

More on gauge of Wannier orbitals: Y-L Wu, A. Bernevig, N. Regnault, PRB (2012)

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'Parallel transport' of phase — 'Polarization' — Fourier transform

Berry connection indicates change of phase due to displacement in BZ

ensures periodicity of WF in $k_y \rightarrow k_y + 2\pi$

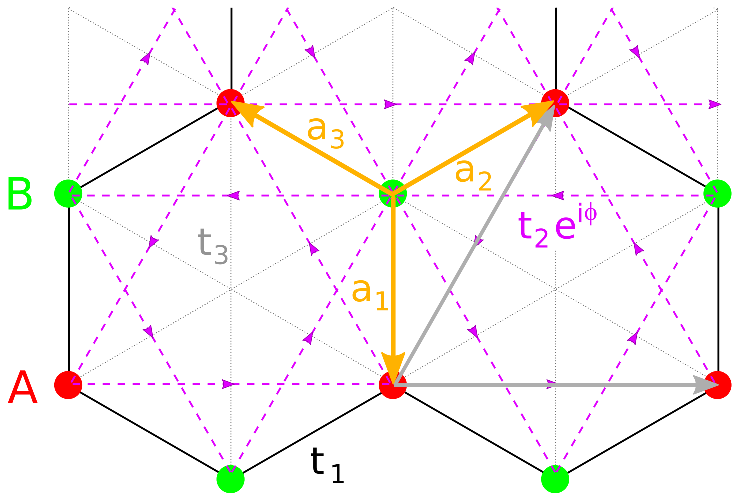
- or, more simply we can think of the Wannier states as the eigenstates of the position operator

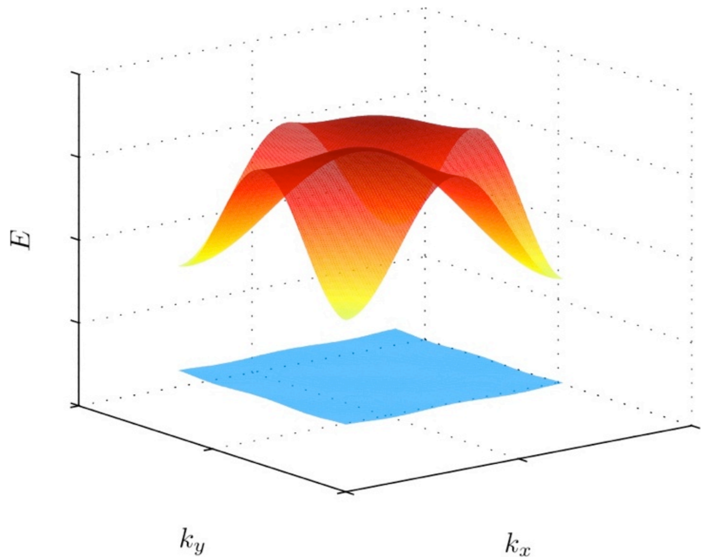
$$\hat{X}^{cg} = \lim_{q_x \to 0} \frac{1}{i} \frac{\partial}{\partial q_x} \bar{\rho}_{q_x} \quad \Rightarrow \quad \hat{X}^{cg}|W(x,k_y)\rangle = [x - \theta(k_y)/2\pi]|W(x,k_y)\rangle$$

- role of polarization: displacement of centre of mass of the Wannier state

$$\theta(k_y) = \int_0^{2\pi} \mathcal{A}_x(p_x,k_y)dp_x$$

An example: The Haldane Model





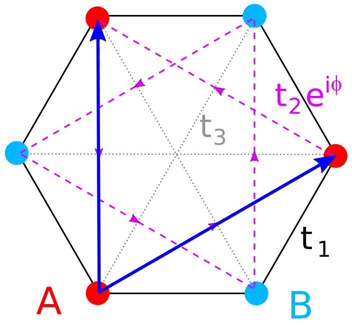
$$\mathcal{H} = -t_1 \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} (\hat{a}_{\mathbf{r}}^\dagger \hat{a}_{\mathbf{r}} + h.c.) - t_2 \sum_{\langle\langle \mathbf{r}\mathbf{r}' \rangle\rangle} (\hat{a}_{\mathbf{r}}^\dagger \hat{a}_{\mathbf{r}} e^{i\phi_{\mathbf{r}\mathbf{r}'}} + h.c.)$$
$$- t_3 \sum_{\langle\langle\langle \mathbf{r}\mathbf{r}' \rangle\rangle\rangle} (\hat{a}_{\mathbf{r}}^\dagger \hat{a}_{\mathbf{r}} + h.c.) + \frac{U}{2} \sum_{\mathbf{r}} \hat{n}_{\mathbf{r}} (\hat{n}_{\mathbf{r}} - 1)$$

- tight binding model on hexagonal lattice
- with fine-tuned hopping parameters: obtain flat lower band

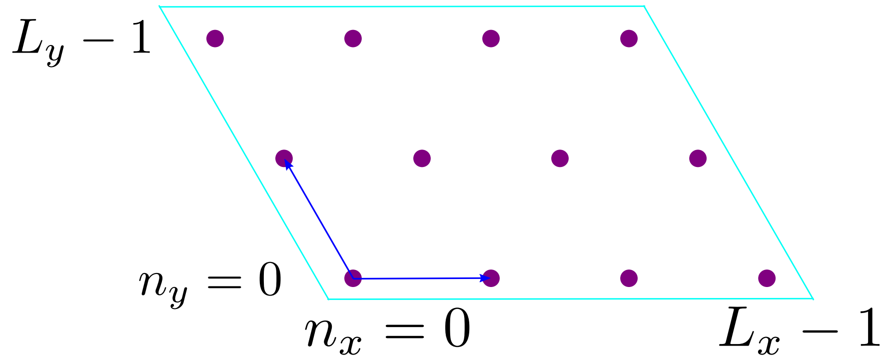
$$t_1 = 1,\ t_2 = 0.60,\ t_2 = -0.58 \text{ and } \phi = 0.4\pi$$

Conventions for numerical evaluation

Real Space



Reciprocal Space



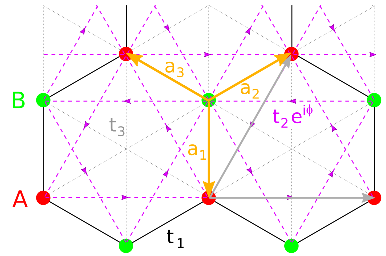
$$\mathbf{v}_1 = \sin(\gamma)\mathbf{e}_x + \cos(\gamma)\mathbf{e}_y$$
$$\mathbf{v}_2 = \mathbf{e}_y$$

$$\mathbf{G}_1 = 2\pi\mathbf{e}_x / L_1 \sin(\gamma)$$
$$\mathbf{G}_2 = 2\pi[-\cot(\gamma)\mathbf{e}_x + \mathbf{e}_y]/L_2$$

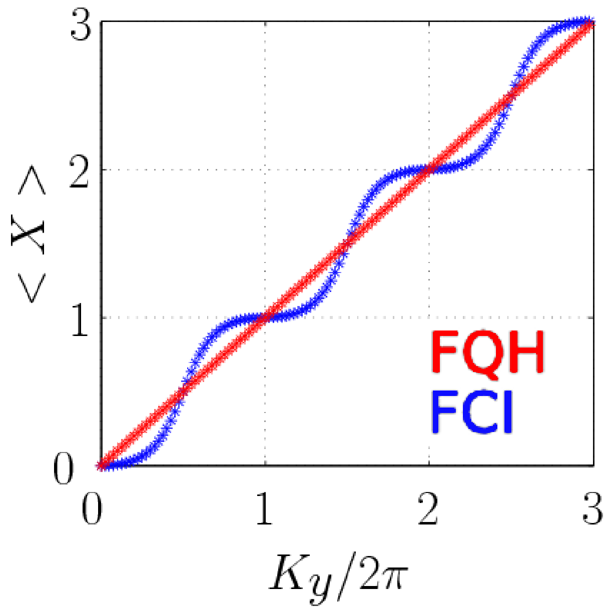
A few remarks:

- choose 'periodic' gauge with Bloch functions: $u^n_\beta(\mathbf{k} + L_i\mathbf{G_i}) = u^n_\beta(\mathbf{k})$
- use discretized Berry connection $A^n_x(q_1, q_2) = \Im \log\left[u^{n*}_\alpha(q_1, q_2)u^n_\alpha(q_1+1, q_2)\right]$
- discretize its integrals by the rectangle rule $\int_0^{k_x} \mathcal{A}_x(p_x, k_y)dp_x \to \sum_{\tilde{q}_1=0}^{q_1(k_x)} A^n_x(\tilde{q}_1, q_2)$

Qi's Mapping



- Can introduce a canonical order of states with monotonously increasing position:



$$k_y = 2\pi n_y / L_y$$

$$\boxed{K_y} = k_y + 2\pi x = \boxed{2\pi j / L_y}$$

$$j = n_y + L_y x = 0, 1, ..., N_\phi - 1$$

- Increase in position for $k_y \rightarrow k_y + 2\pi$ = Chern-number C, as

$$\frac{\partial}{\partial k_y} \langle \hat{X}^{cg} \rangle |_x = -\frac{1}{2\pi} \frac{\partial \theta(k_y)}{\partial k_y} = \int_0^{2\pi} \mathcal{B}(p_x, k_y) dp_x$$

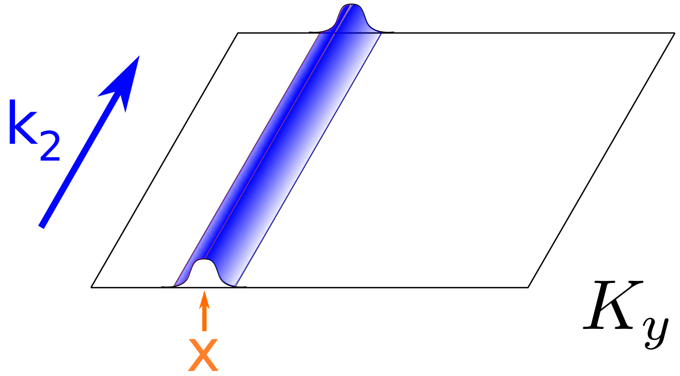
More on Wannier states:

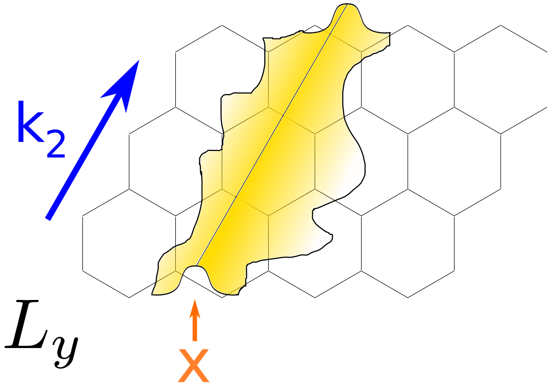
Y-L Wu, A. Bernevig, N. Regnault, PRB (2012)

Z. Liu & E. Bergholtz, PRB 2013

Case study: Bosons with contact interactions

$$\mathcal{H}_{\text{int}} \propto \sum_{i<j} \delta(r_i - r_j)$$





$$K_y = k_y + 2\pi x = 2\pi j / L_y$$

$$k_y^{\max} = N_\phi - 1$$

expand Hamiltonian in single-particle orbitals
(finite size, periodic boundary conditions)

$$K_y^{\max} = L_x \times L_y - 1$$

$$\mathcal{H} = \sum_{j_1, j_2, j_3, j_4} V_{j_1 j_2; j_3 j_4} \hat{c}^\dagger_{j_1} \hat{c}^\dagger_{j_2} \hat{c}_{j_3} \hat{c}_{j_4}$$

- Landau level momentum conserved:

$$V_{j_1 j_2; j_3 j_4} \propto \delta_{j_1+j_2,\, j_3+j_4}$$

- Linearized momentum

$$V_{j_1 j_2; j_3 j_4} \propto \delta^{\mod L_x}_{j_1+j_2,\, j_3+j_4}$$

T. Scaffidi, GM, PRL (2012)
[arxiv:1207.3539]

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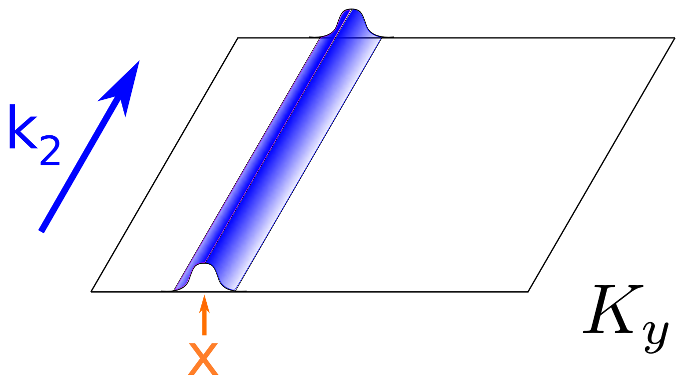

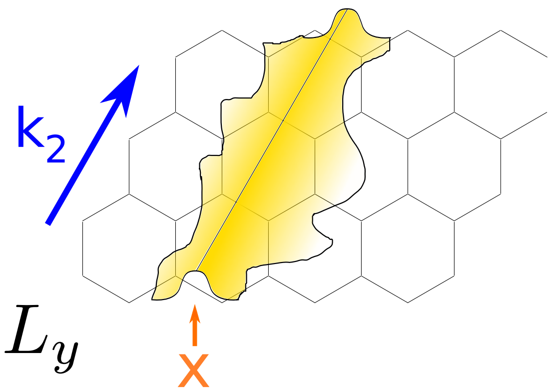

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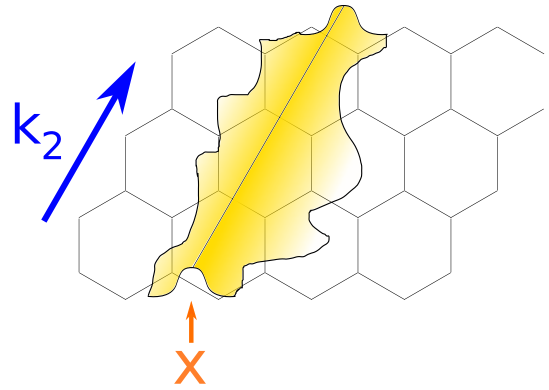
T. Scaffidi, GM, PRL (2012)
[arxiv:1207.3539]

Different conservation laws
→ Problems live in different Hilbert spaces

Matrix elements in the Wannier basis

$$\mathcal{H}_{\text{int}} \propto \sum_{i<j} \delta(r_i - r_j)$$

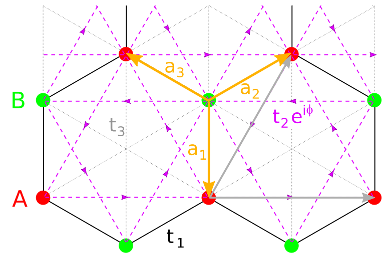
$$K_y = k_y + 2\pi x = 2\pi j / L_y$$



$$\mathcal{H}^{FCI} = \sum_{\substack{k_{y1},k_{y2},k_{y3},k_{y4} \\ x_1,x_2,x_3,x_4 \\ k_{y1}+k_{y2}=k_{y3}+k_{y4}}} \hat{c}^\dagger_{W(k_{y1},x_1)} \hat{c}^\dagger_{W(k_{y2},x_2)} \hat{c}_{W(k_{y3},x_3)} \hat{c}_{W(k_{y4},x_4)}$$

$$V_{j_1 j_2; j_3 j_4} \left\{ \begin{array}{l} \displaystyle\sum_{\substack{k_{x1},k_{x2},k_{x3},k_{x4} \\ k_{x1}+k_{x2}=k_{x3}+k_{x4}}} f^{*(x_1,k_{y_1})}_{k_{x1}} f^{*(x_2,k_{y2})}_{k_{x2}} f^{(x_3,k_{y3})}_{k_{x3}} f^{(x4,k_{y4})}_{k_{x4}} \\ \displaystyle\sum_{a=A,B} u^{*a}_{\alpha_0}(\mathbf{k_1}) u^{*a}_{\alpha_0}(\mathbf{k_2}) u^{a}_{\alpha_0}(\mathbf{k_3}) u^{a}_{\alpha_0}(\mathbf{k_4}) \end{array} \right.$$

Case study: Bosons with contact interactions

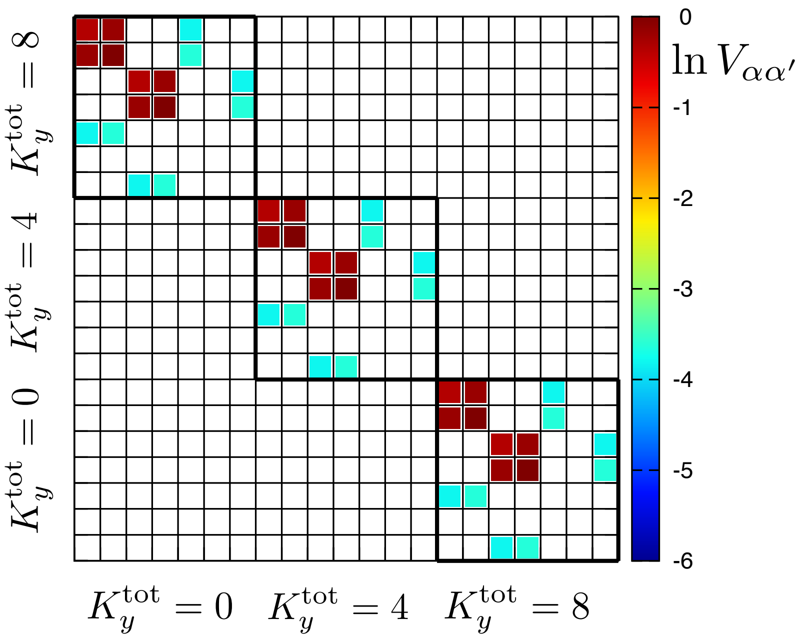


Th. Scaffidi & GM, Phys. Rev. Lett. **109**, 246805 (2012) [arxiv:1207.3539]

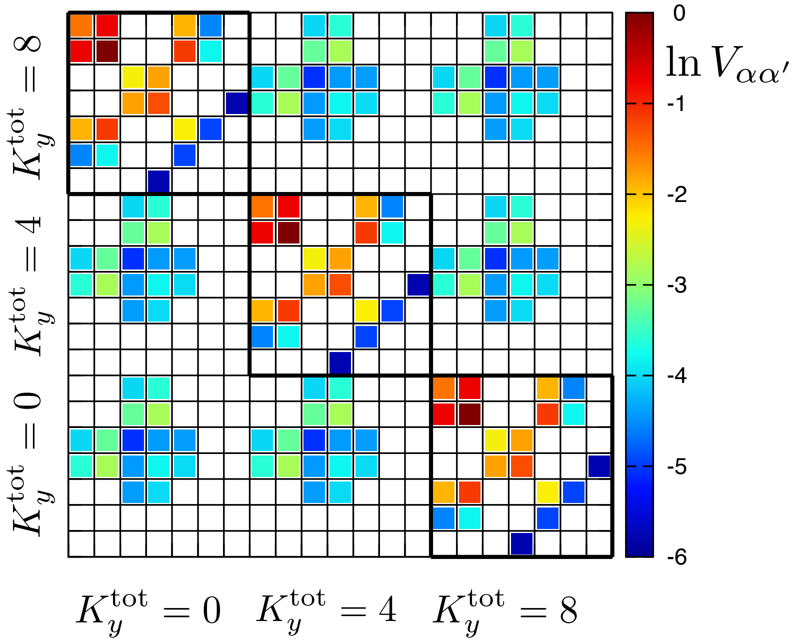
- Magnitude of two-body matrix elements for delta interactions in the Haldane model

$$V(\vec{r}_i - \vec{r}_j) \propto \delta(\vec{r}_i - \vec{r}_j)$$

FQHE



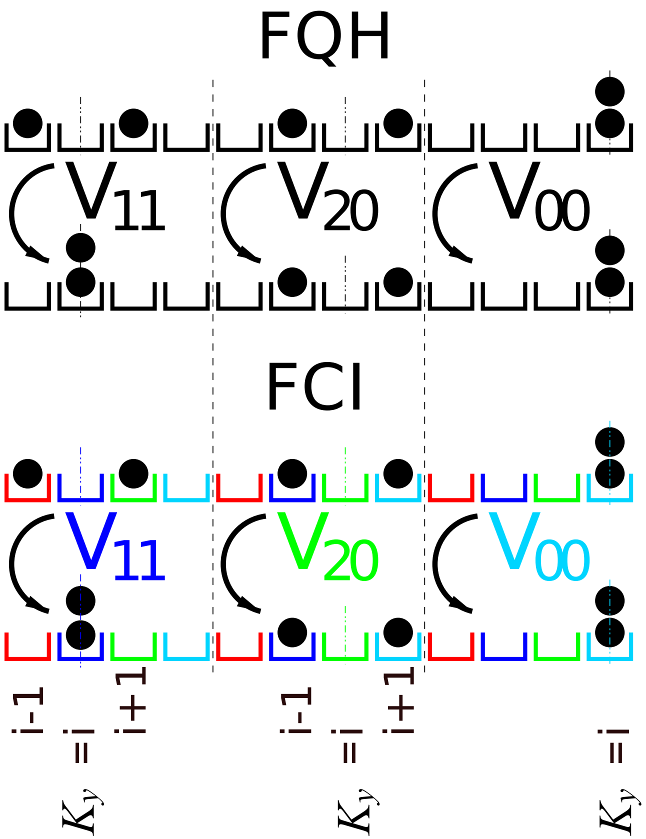
FCI

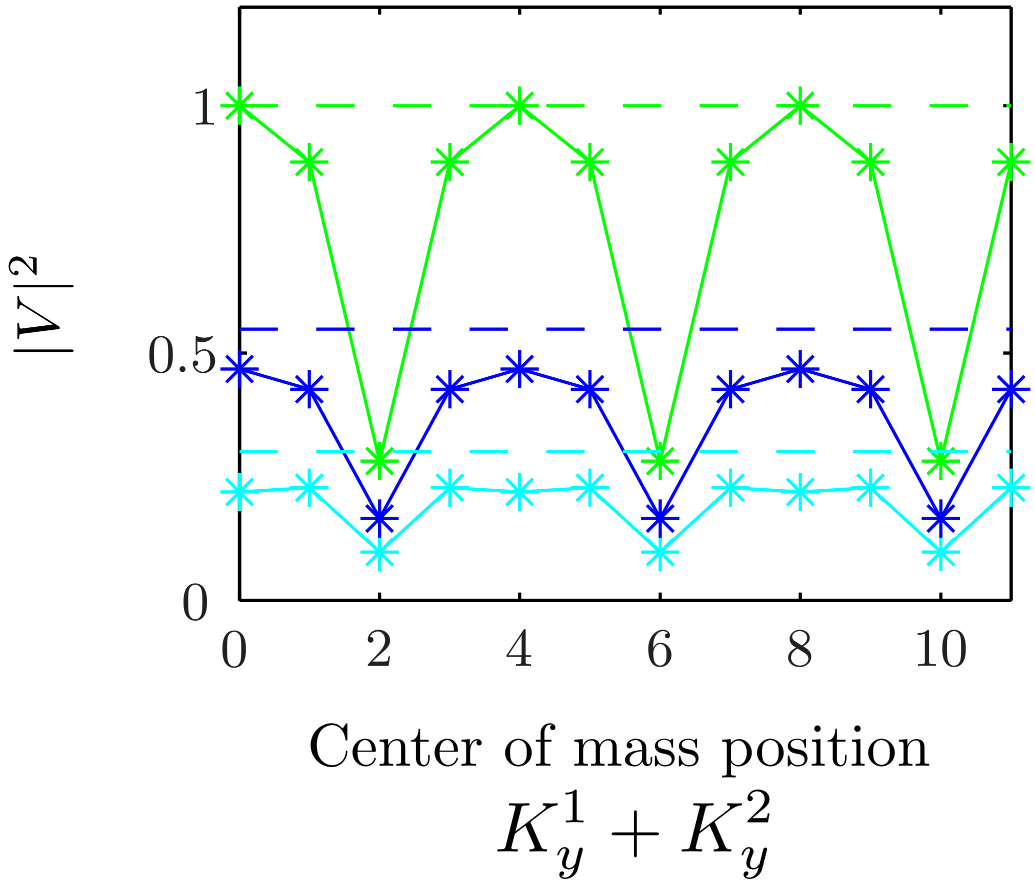


- System shown: two-body interactions for $L_x \times L_y = 3 \times 4$
- Matrix elements differ in magnitude, but overall similarities are present
- Different block-structure due to non-conservation of linearized momentum K_y
- Lack of translational invariance of matrix elements in momentum space

Reduced translational invariance in K_y

- A closer look at some short range hopping processes





- for FCI: hopping amplitudes depend on position of centre of mass / K_y

Interpolating in the Wannier basis

- Can write both states in single Hilbert space with the same overall structure (indexed by K_y) and study the low-lying spectrum numerically (exact diagonalization)
- Can study adiabatic deformations from the FQHE to a fractionally filled Chern band

$$\mathcal{H}(x) = \frac{\Delta_{\mathrm{FCI}}}{\Delta_{\mathrm{FQHE}}}(1-x)\mathcal{H}^{\mathrm{FQHE}} + x\mathcal{H}^{\mathrm{FCI}}$$

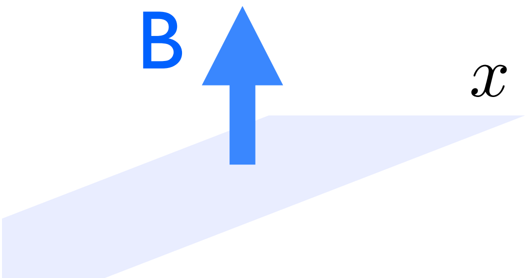
- Here: look at half-filled band for bosons

FQHE of Bosons at $\nu = 1/2$

Half filled band of the (flattened) Haldane-model

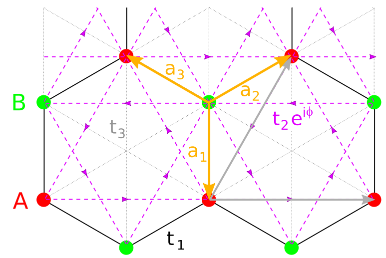
Laughlin state

Same topological phase?



$x = 0$

$x = 1$



Th. Scaffidi & GM, Phys. Rev. Lett. (2012)

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Evaluating the accuracy of the Wannier states: Overlaps

- Can write both states in single Hilbert space with the same overall structure (indexed by K_y) and study the low-lying spectrum numerically (exact diagonalization)
- Can study adiabatic deformations from the FQHE to a fractionally filled Chern band

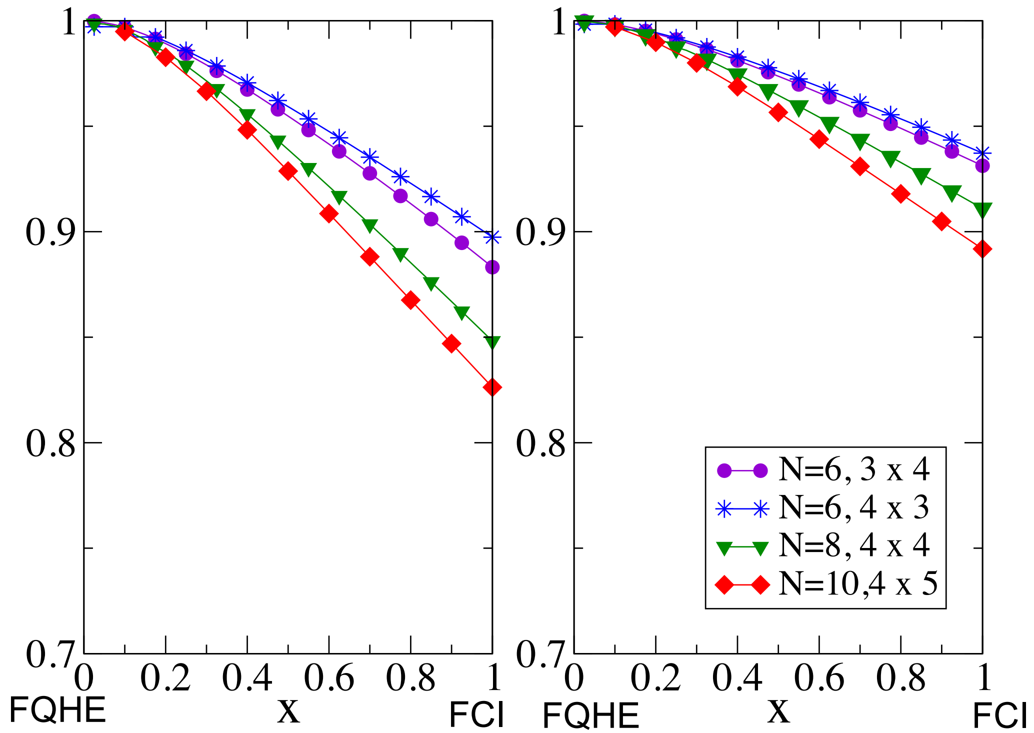
$$\mathcal{H}(x) = \frac{\Delta_{\mathrm{FCI}}}{\Delta_{\mathrm{FQHE}}}(1-x)\mathcal{H}^{\mathrm{FQHE}} + x\mathcal{H}^{\mathrm{FCI}}$$

Overlap

$|\langle\Psi(x)|\Phi\rangle|^2$

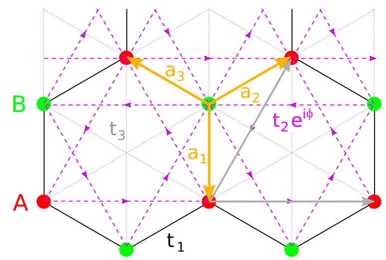
Weight in GS sector

$|\mathcal{P}_{k_{\mathrm{GS}}}|\Psi\rangle|^2$



Bosons at

$\nu = 1/2$

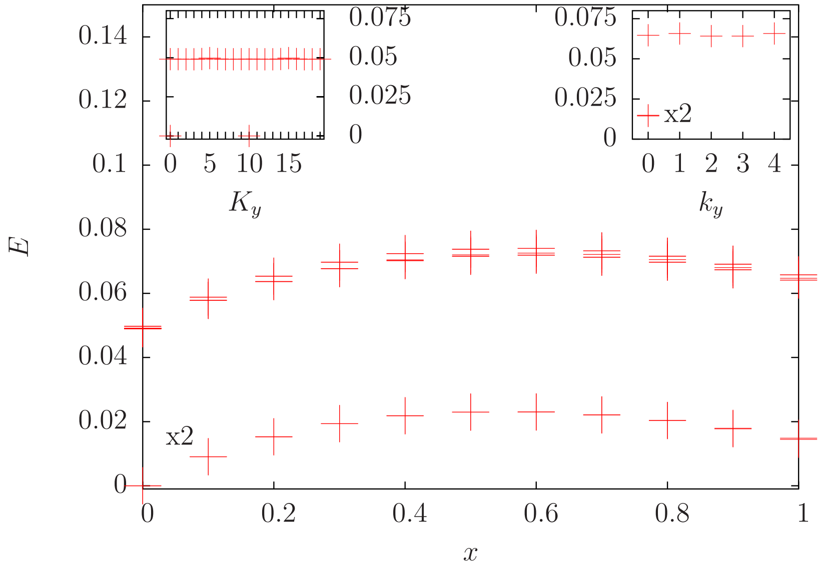


Th. Scaffidi & GM, Phys. Rev. Lett. (2012) [arxiv:1207.3539]

Adiabatic continuation in the Wannier basis

- Spectrum for N=10:

FQHE FCI



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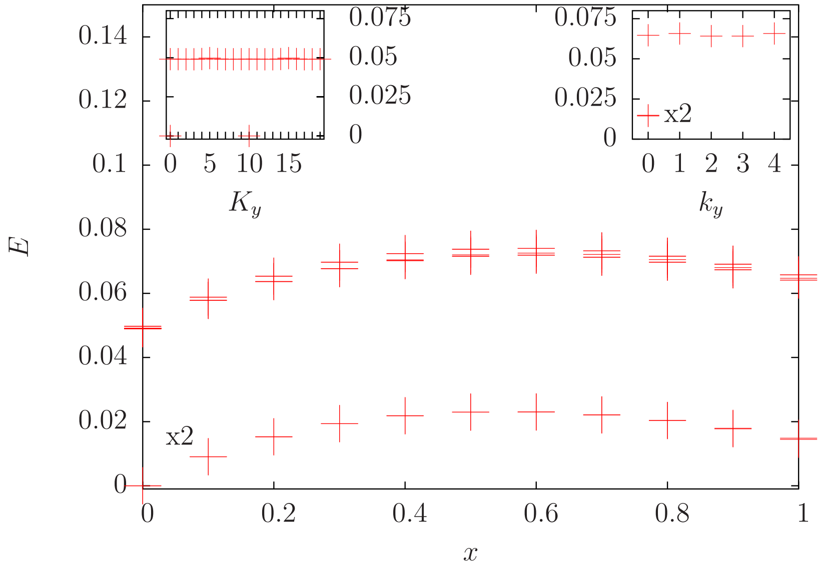
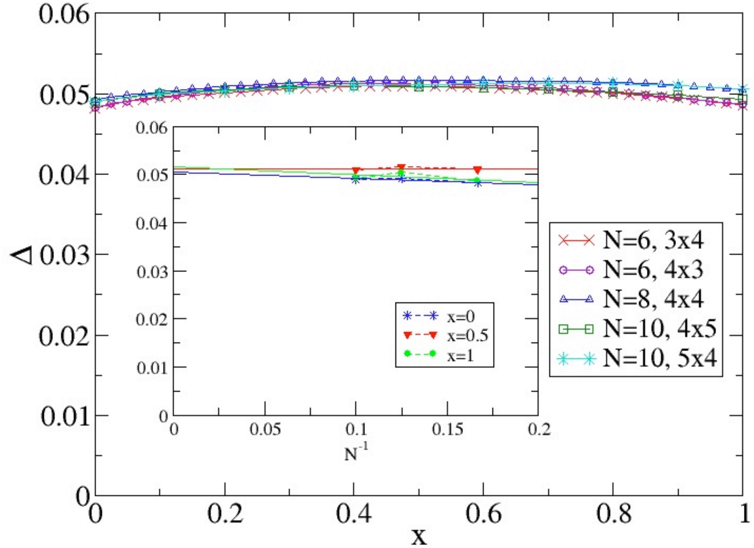


- Gap for different system sizes & aspect ratios:

FQHE FCI



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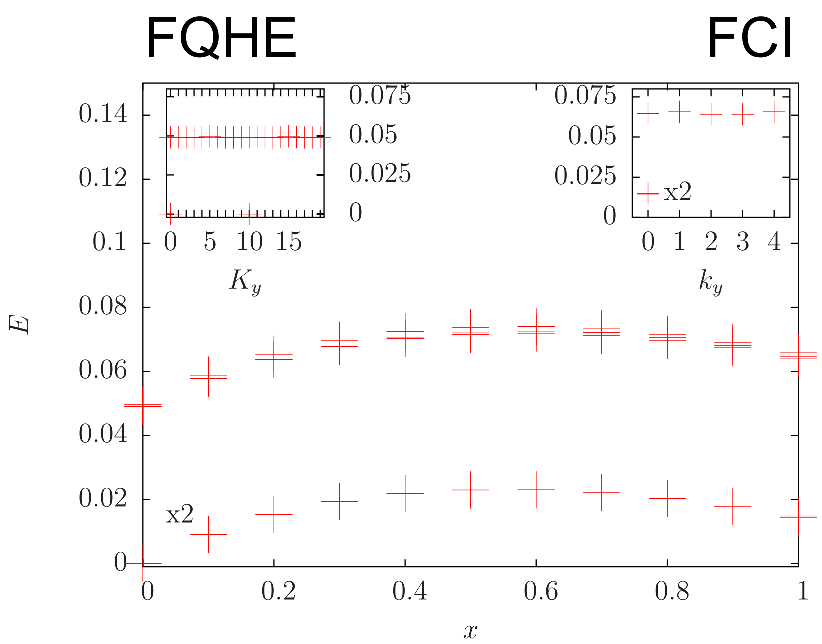

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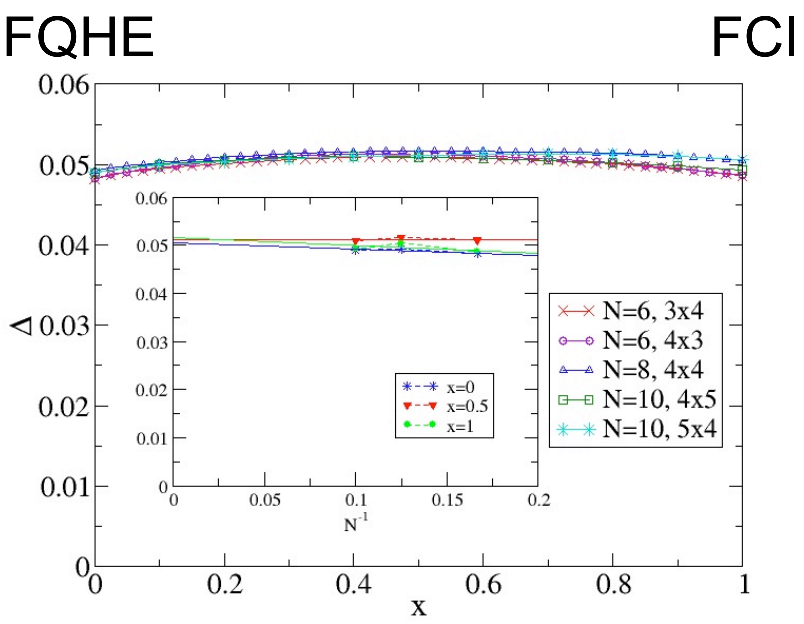


- We confirm the Laughlin state is adiabatically connected to the groundstate of the half-filled topological flat band of the Haldane model

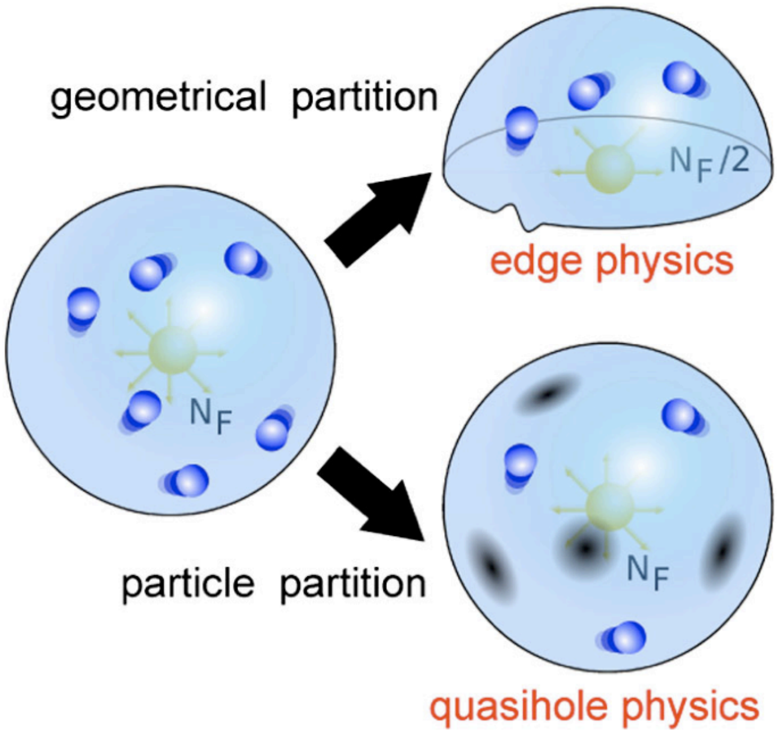
- Clean extrapolation to the thermodynamic limit - (unlike overlaps)

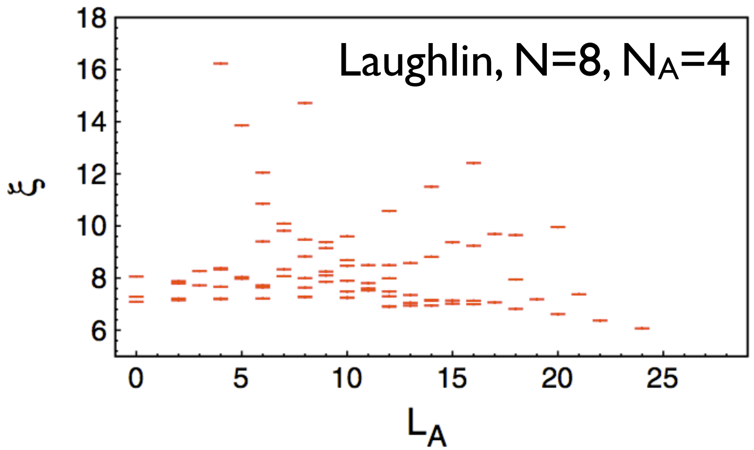
Th. Scaffidi & GM, Phys. Rev. Lett. (2012) [arxiv:1207.3539]

Entanglement spectra and quasiparticle excitations

• Entanglement spectrum: arises from Schmidt decomposition of ground state into two groups A, B
=> Schmidt eigenvalues ξ plotted over quantum numbers for symmetries within each block

$$|\Psi\rangle = \sum_{\varpi} \sum_{i} e^{-\xi_{\varpi,i}/2} |\Psi^{A}_{\varpi,i}\rangle \otimes |\Psi^{B}_{\varpi,i}\rangle$$





Dominant (universal) eigenvalues of PES yield count of excited states - and their wavefunctions - from groundstate wavefunction only!

credit: A. Sterdyniak et al. PRL 2011

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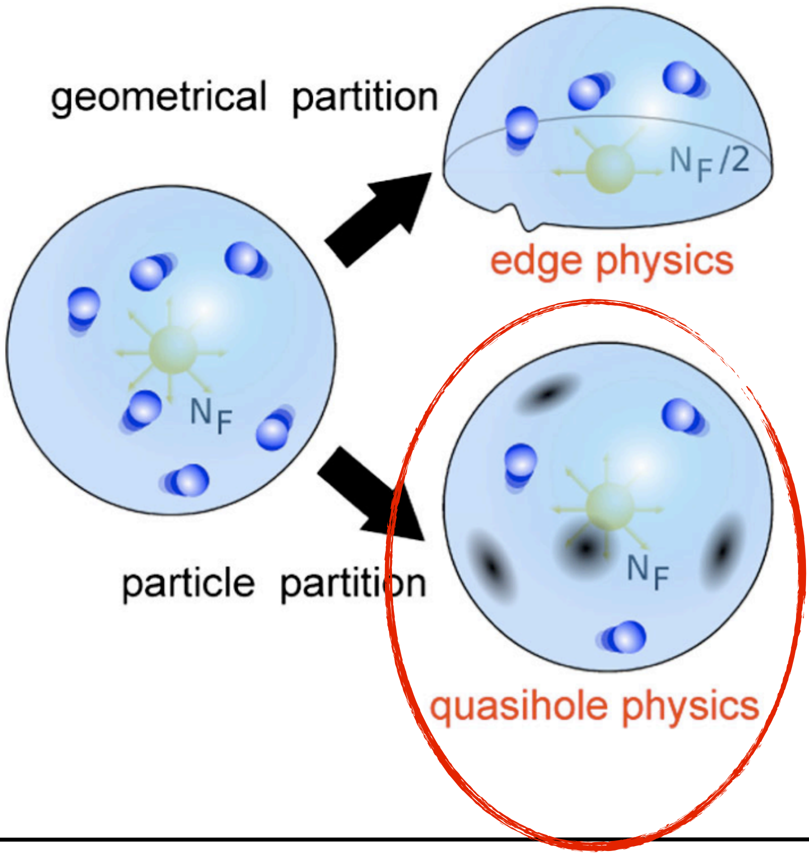
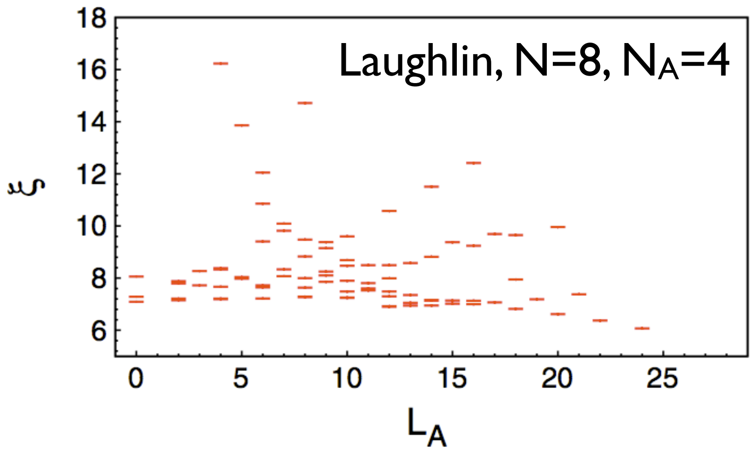




Dominant (universal) eigenvalues of PES yield count of excited states - and their wavefunctions - from groundstate wavefunction only!

credit: A. Sterdyniak et al. PRL 2011

FCI: Adiabatic continuation of the entanglement spectrum

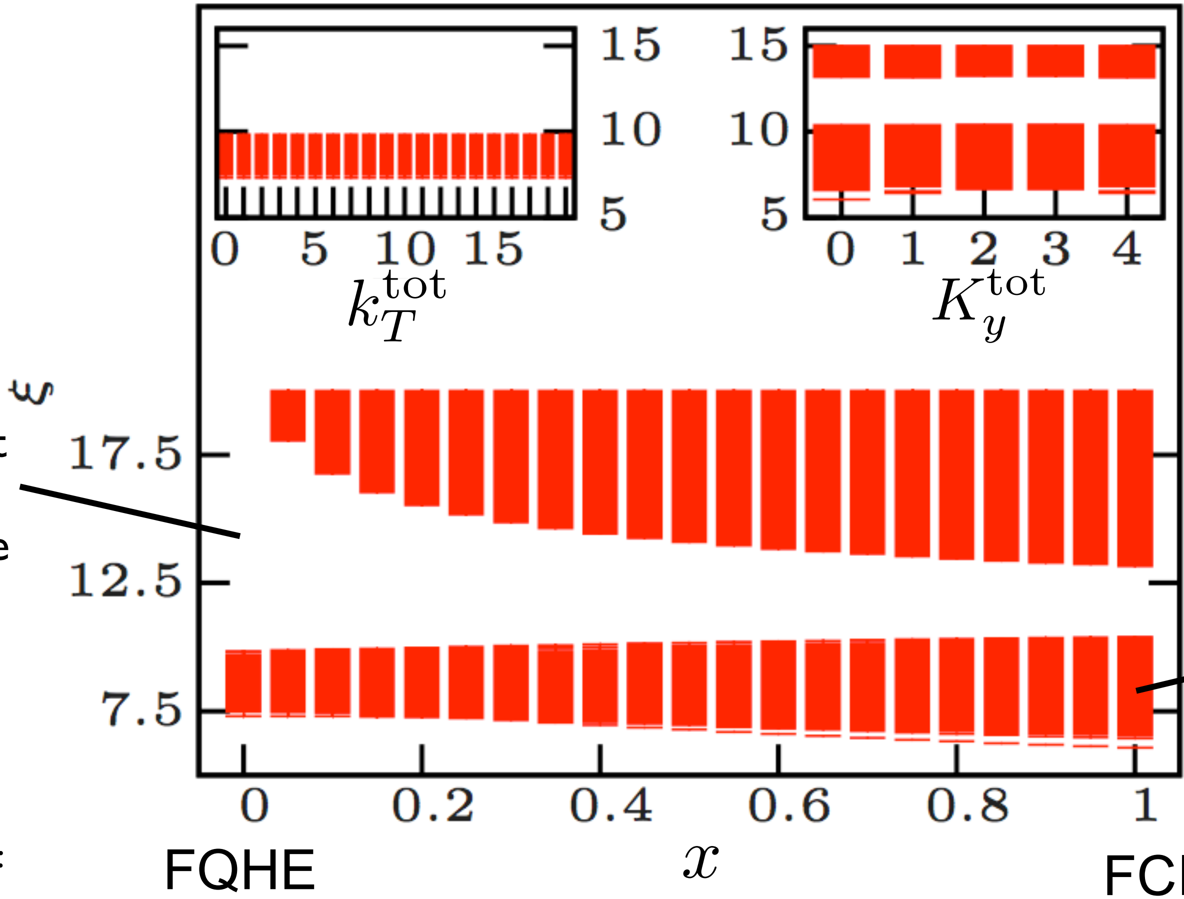
Total #eigenvalues below entanglement gap
= 4x(201 + 200 + 200 + 200 + 200)

Total #eigenvalues below entanglement gap
= 804 + 800 + 800 + 800 + 800

$N = 10$

$N_A = 5$

'Infinite' entanglement gap for pure Laughlin state



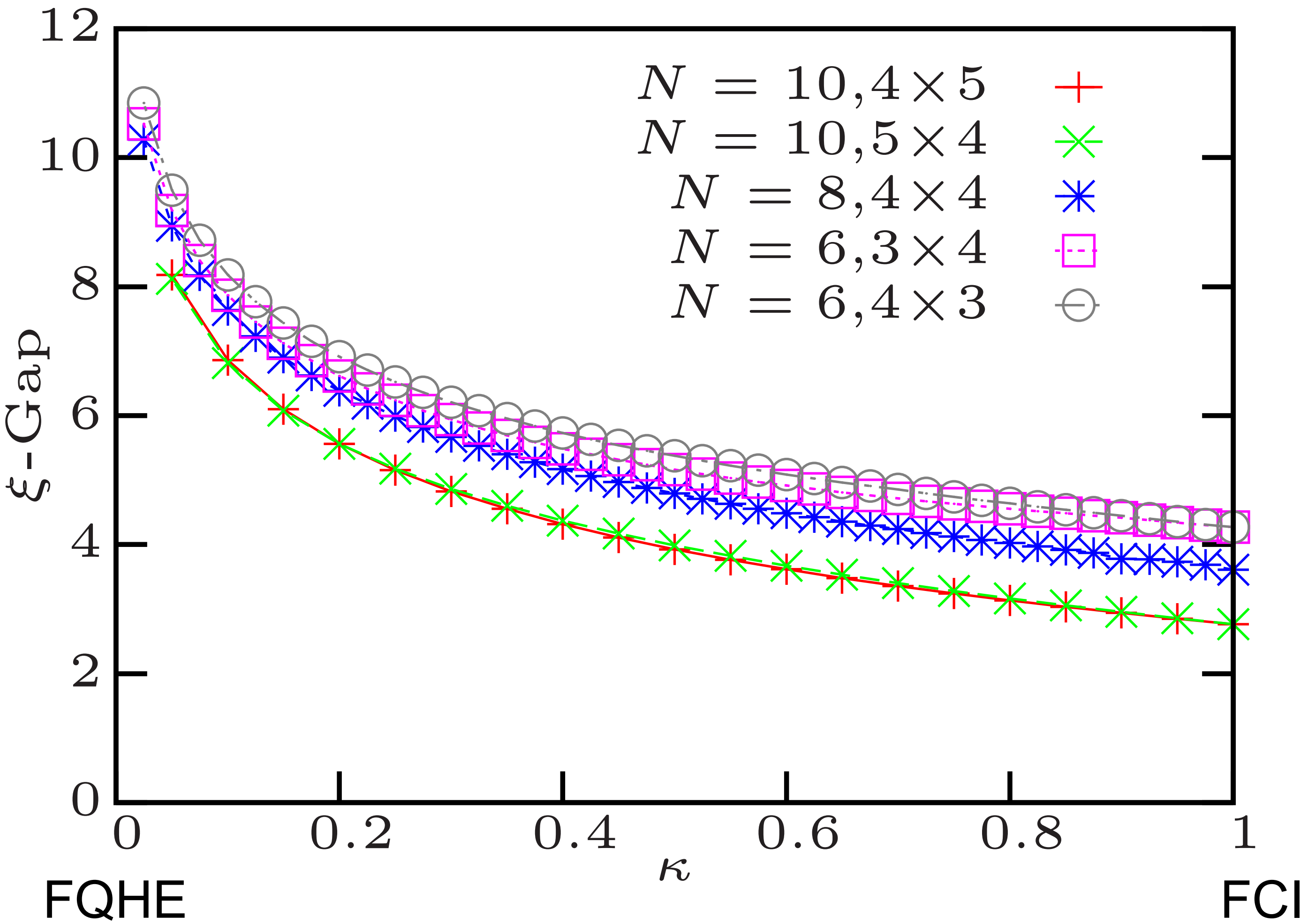
Same number of states for all x

Dictionary: FQHE FCI

$$|\Psi\rangle = \sum_{\varpi} \sum_{i} e^{-\xi_{\varpi,i}/2} |\Psi^A_{\varpi,i}\rangle \otimes |\Psi^B_{\varpi,i}\rangle$$

Th. Scaffidi & GM, Phys. Rev. Lett. (2012) [arxiv:1207.3539]

Finite size behaviour of entanglement gap



- The entanglement gap remains open for all values of the interpolation parameter k
- Finite size scaling behaviour encouraging, but analytic dependency on system size unknown

Conclusions: FCI wavefunctions from Wannier states

- Wavefunctions of FCI's in the Wannier basis are similar but not identical to FQH states in the Landau gauge

- We demonstrated the adiabatic continuity of the ground states at $\nu=1/2$ using Qi's mapping between Wannier basis and FQH eigenstates

- FCI wavefunctions not very accurate for the Haldane model (higher overlaps in models with N>2 sublattices)

higher overlaps also by explicit gauge fixing, see:
Wu, Regnault, Bernevig, PRB (2012)

Th. Scaffidi & GM, Phys. Rev. Lett. **109**, 246805 (2012) [arxiv:1207.3539]