Correlated Phases of Bosons in the Flat Lowest Band of the Dice Lattice

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TCM

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Outline

- Introduction: Artificial gauge fields and strongly correlated states on lattices

- Lattice geometry as a key to Hofstadter spectra: square vs dice lattice

- Effective low-energy model of bosons on the dice lattice

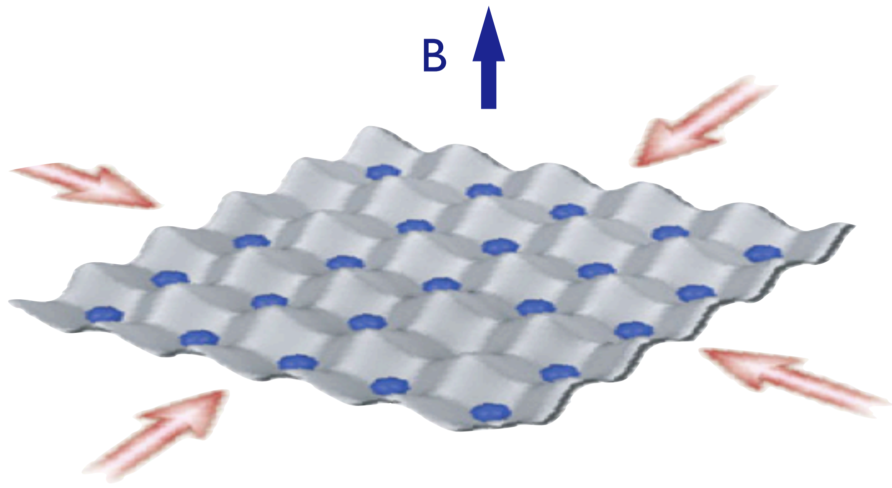
- Many-body phases in the resulting flat-band model:
  - Gross-Pitaevski mean-field approach
  - Numerical study of phases at low particle density

- Some thoughts about localised states in flat bands

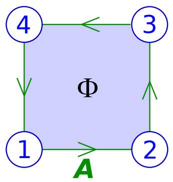
Artificial Gauge Fields on Optical Lattices

experimental realisation:
optical lattice + Raman lasers

$\Rightarrow$ possibility to simulate effect of magnetic field by imprinting phases for hopping (alternatively: Berry phases)



$$\sum_{\square} A_{\alpha\beta} = 2\pi n_{\mathrm{V}}$$



$\Rightarrow$ Bose-Hubbard with a magnetic field ($\rightarrow$ force de Lorentz)

$$\mathcal{H} = -J \sum_{\langle\alpha,\beta\rangle} \left[\hat{b}_\alpha^\dagger \hat{b}_\beta e^{iA_{\alpha\beta}} + h.c.\right] + \frac{1}{2}U \sum_\alpha \hat{n}_\alpha(\hat{n}_\alpha - 1) - \mu \sum_\alpha \hat{n}_\alpha$$

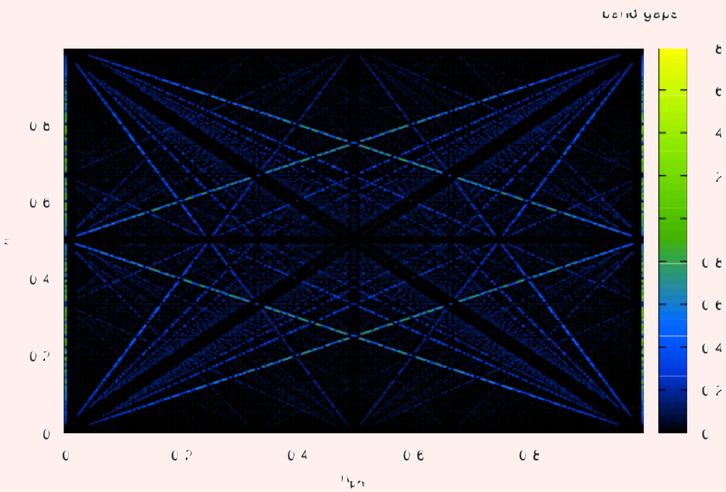
particle density n vortex/flux density n_{V} interaction U/J

J. Dalibard, et al. Rev. Mod. Phys. 83, 1523 (2011)

Many-body states of bosons on the square lattice

Quantum Hall Regime

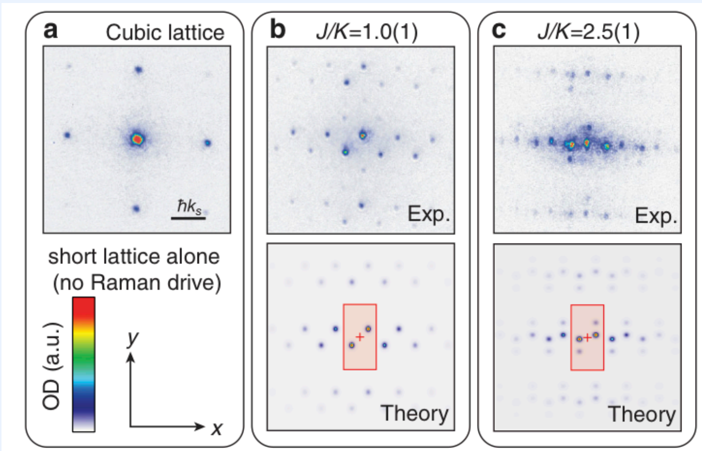
- driven by onsite repulsion U
- promiment states from the continuum limit: Laughlin state, Jain composite fermion states, Moore-Read state
- novel QH states stabilized by the lattice



Sørensen et al. PRL 2006
Palmer & Jaksch, PRL 2006,
GM & N.R. Cooper, PRL 2009

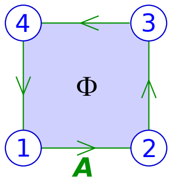
Bose Condensates with Vortex Lattices

- always exist for weak repulsion U
- can survive to large U
- can be thought of as vortex lattices
- states break discrete lattice symmetries
- experimental realization: I. Bloch

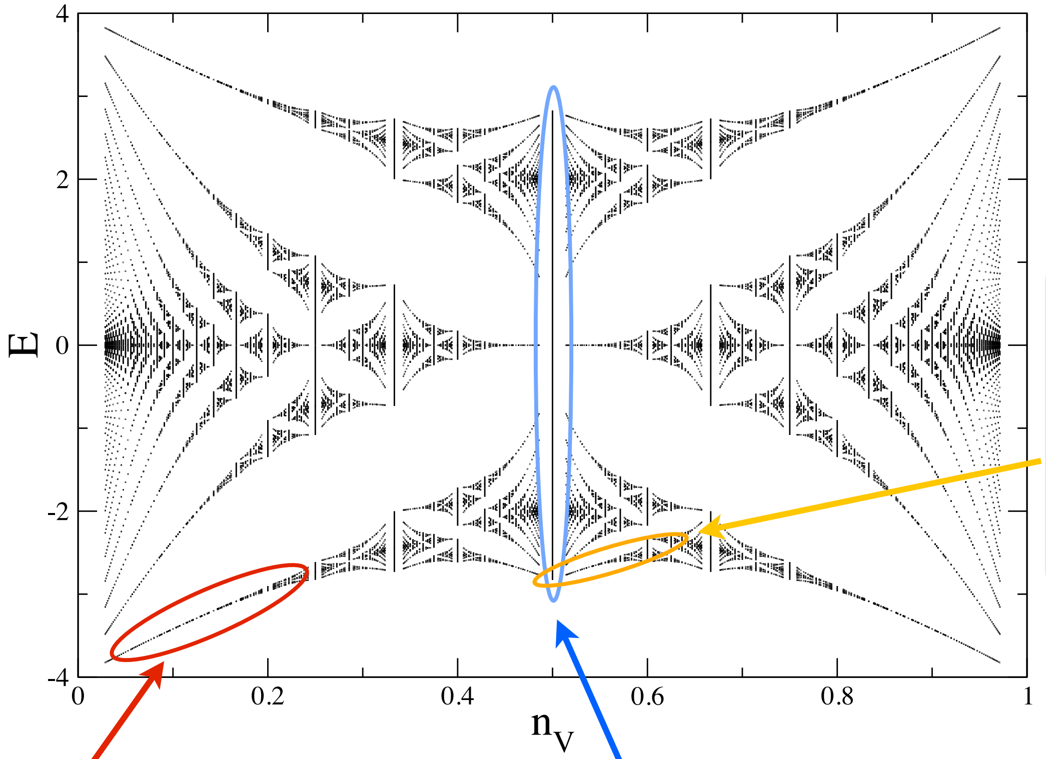


GM & N.R. Cooper, PRA 2010
Đurić & Lee PRB (2010), ...

Single particle spectrum as the hidden key



square lattice at flux density n_V

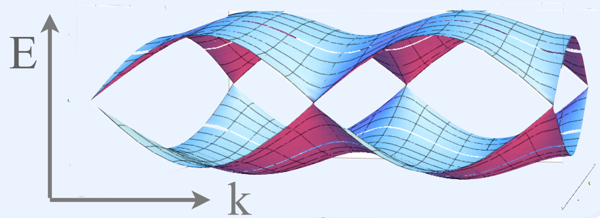


narrow bands with a gap
➜ novel Fractional Quantum Hall states beyond those known in the continuum!

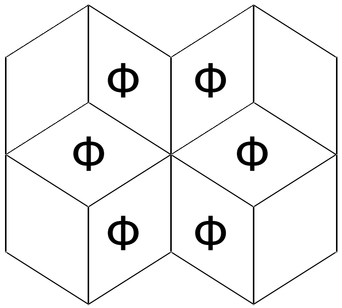
GM & N.R. Cooper, PRL 2009; PRA 2010

narrow band (lowest Landau level)
➜ Fractional Quantum Hall states

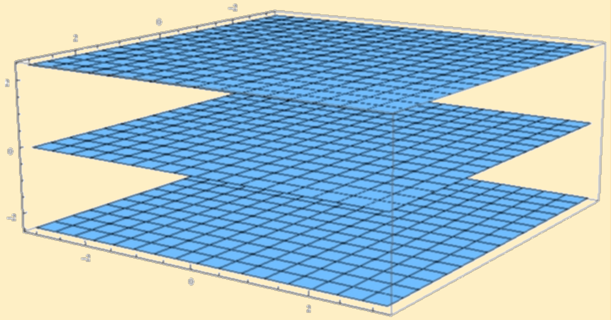
n_V=1/2: wide bands
➜ condensates



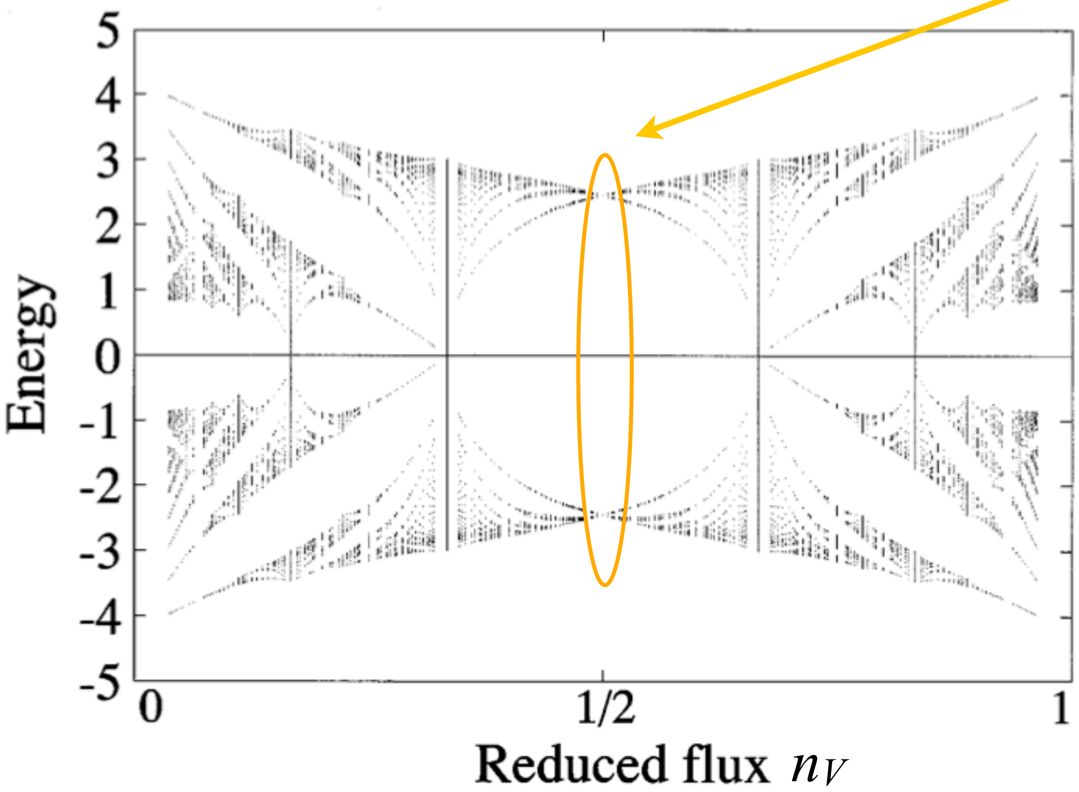
Lattice geometry as the key to Hofstadter spectrum



'dice'-lattice at flux density n_V



$n_V=1/2$: perfectly flat bands!



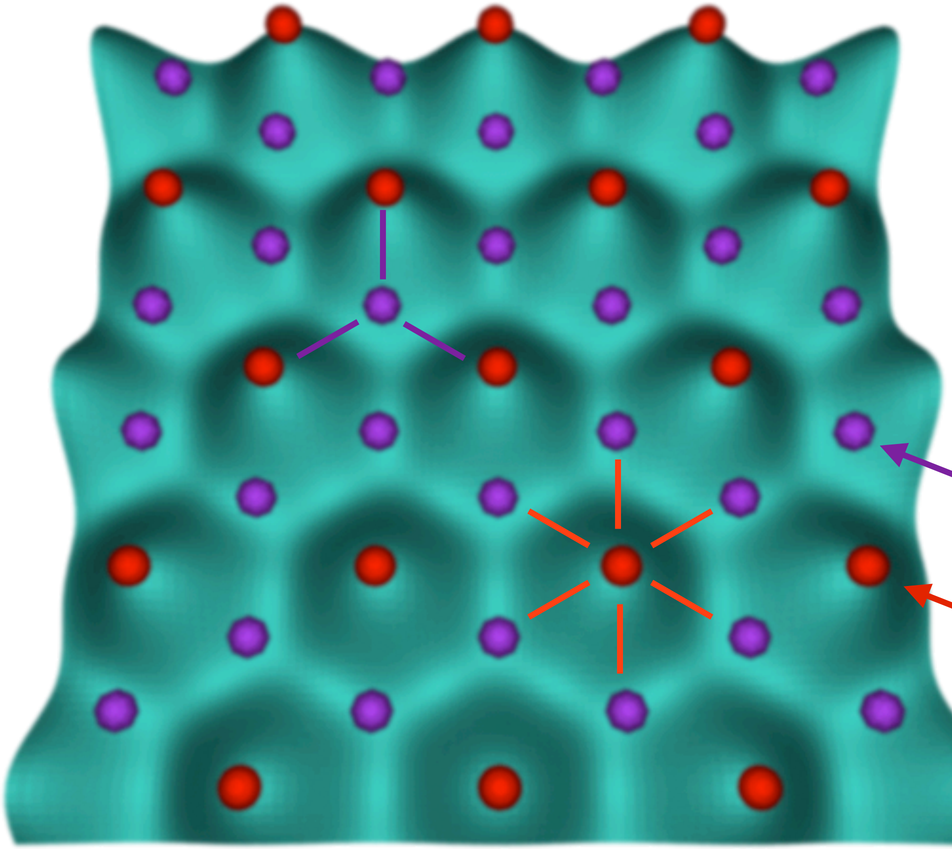
Vidal, Mosseri, Douçot, PRL (1998)

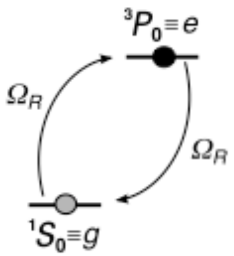
Q1: How to generate this model?

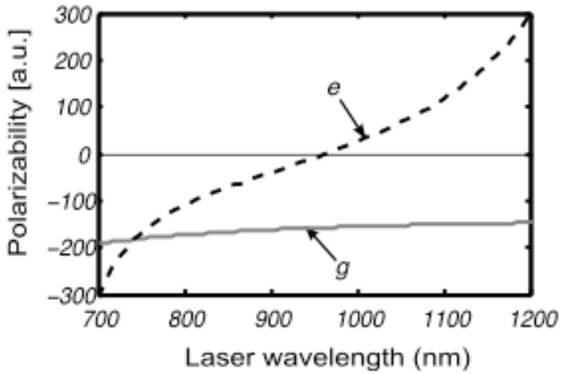
Q2: What are the many-body phases of interacting bosons in this flat band?

Realizing a dice lattice with cold atoms

- three standing waves at 120° at antimagic wavelength, e.g. for Yb (mutually incoherent)





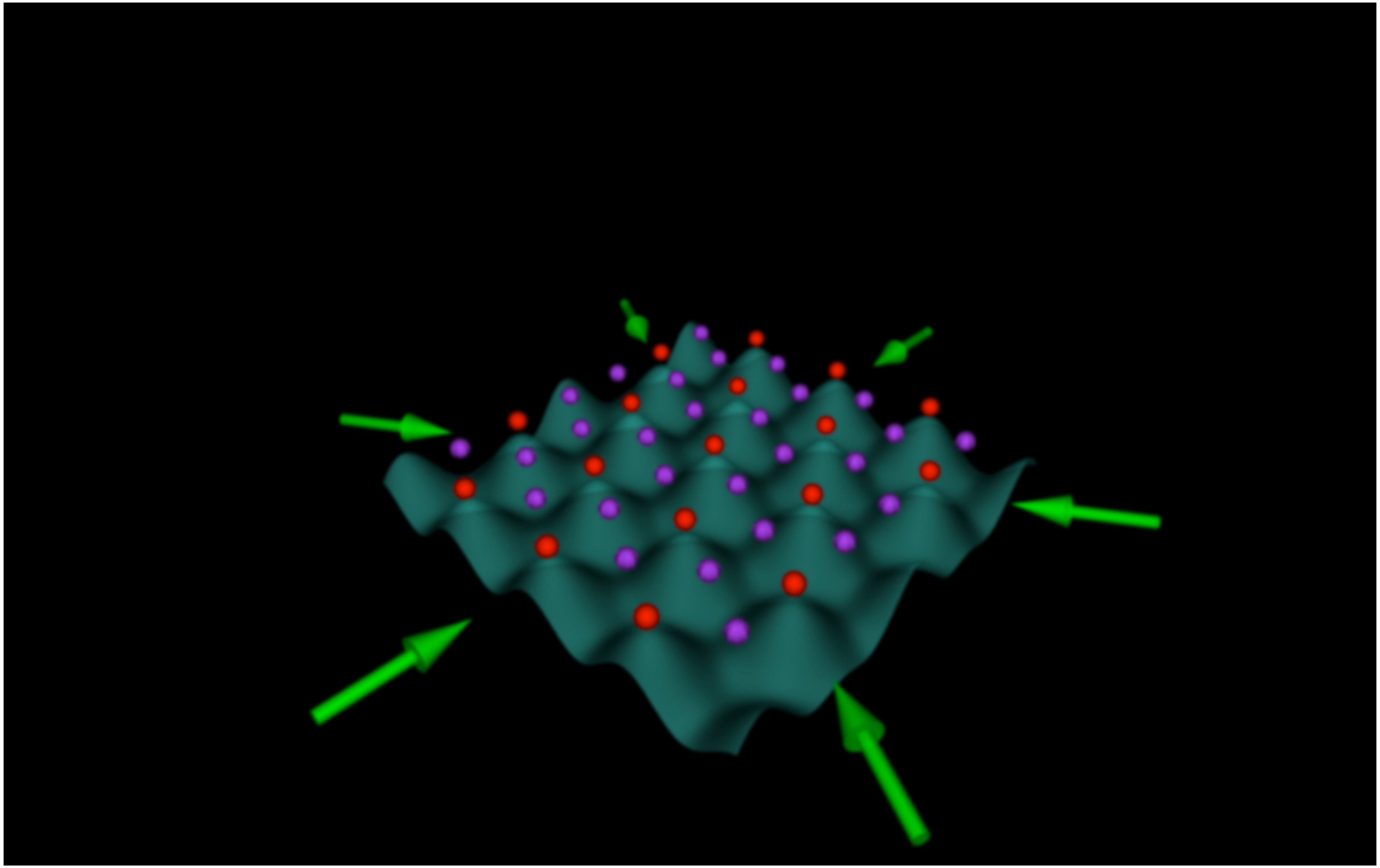


Gerbier & Dalibard, NJP 2010

- use Raman lasers to drive hoppings between neighboring sites of different species ➜ dice-lattice geometry

Realizing a dice lattice with cold atoms

- three standing waves at 120° at antimagic wavelength, e.g. for Yb (mutually incoherent)



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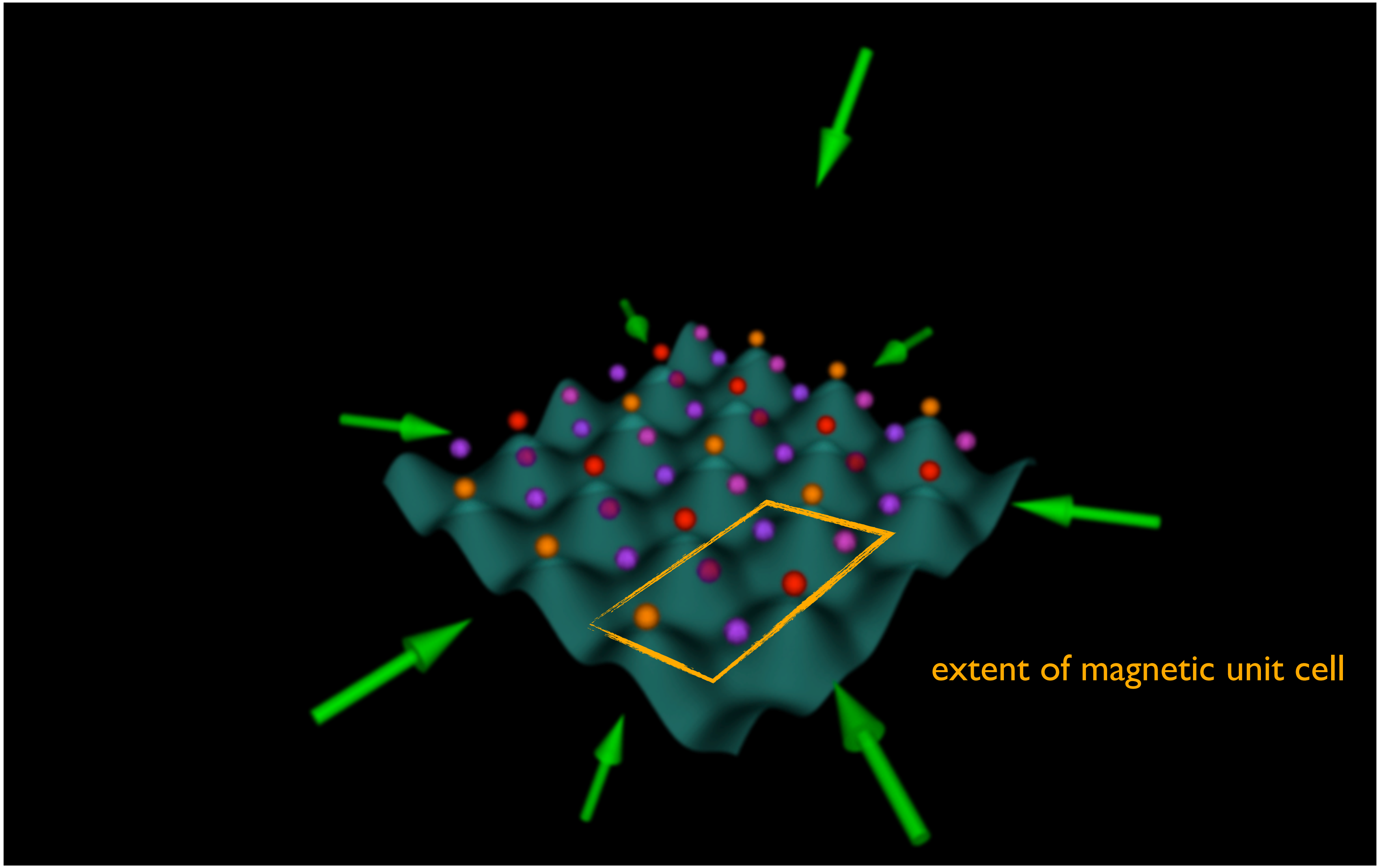
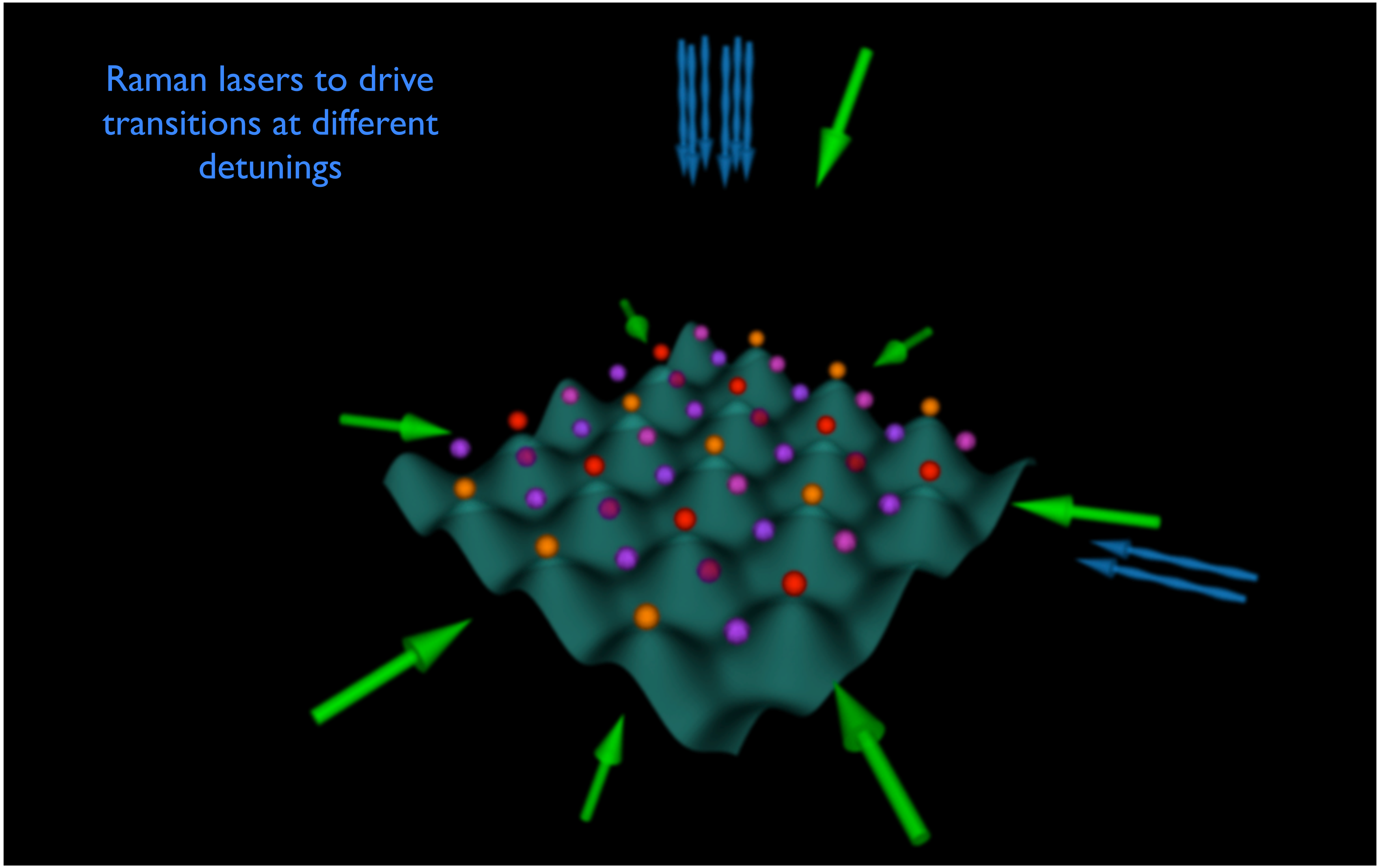
Realizing a dice lattice with cold atoms

- one more laser at antimagic wavelength to break symmetry within magnetic unit cell



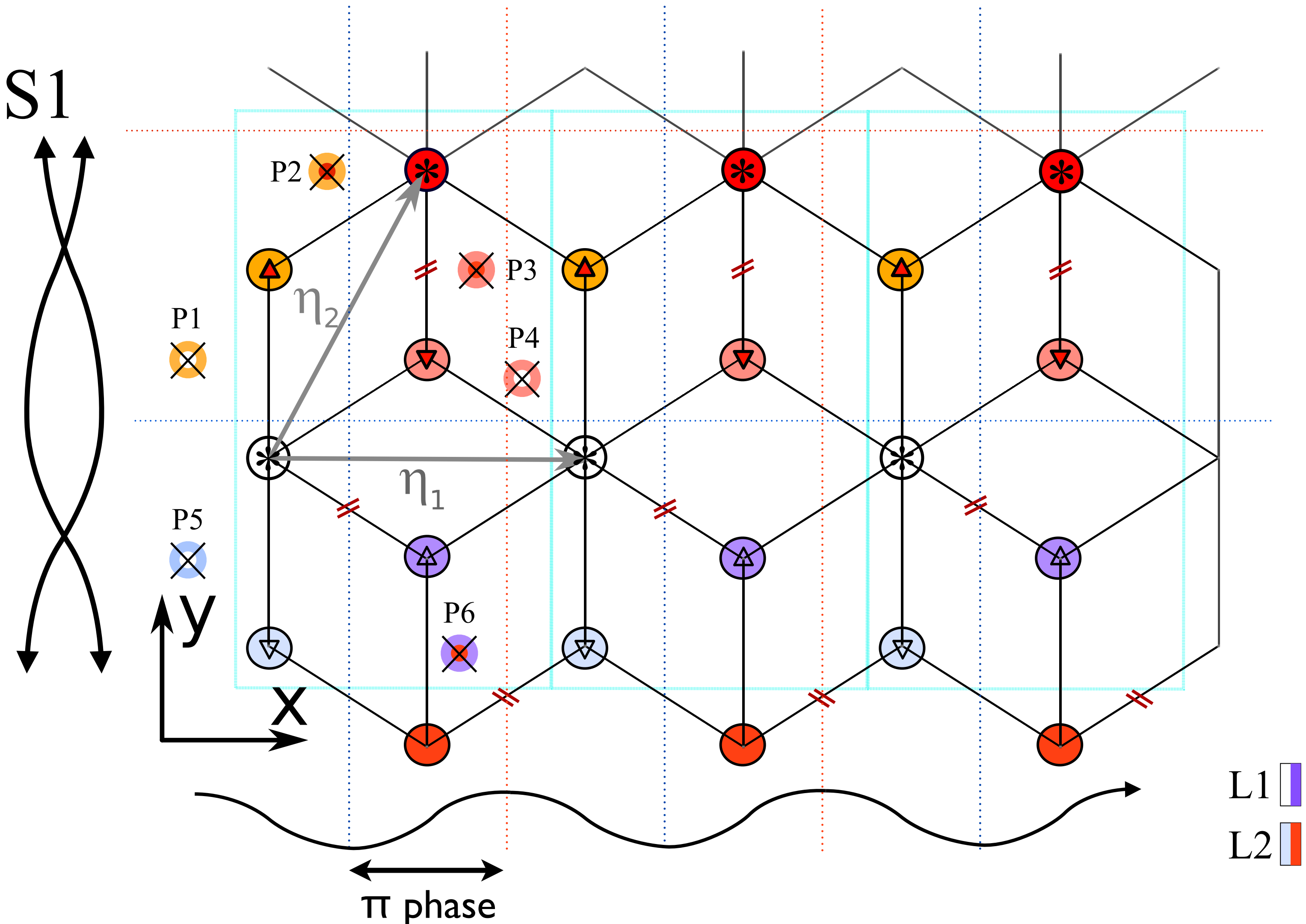
Realizing a dice lattice with cold atoms

- a total of 8 Raman lasers is required to drive the transitions between different sublattices



Raman transitions for the Dice lattice set-up

Choice of gauge $A_{\mu j}$: implementation of Raman driven transitions

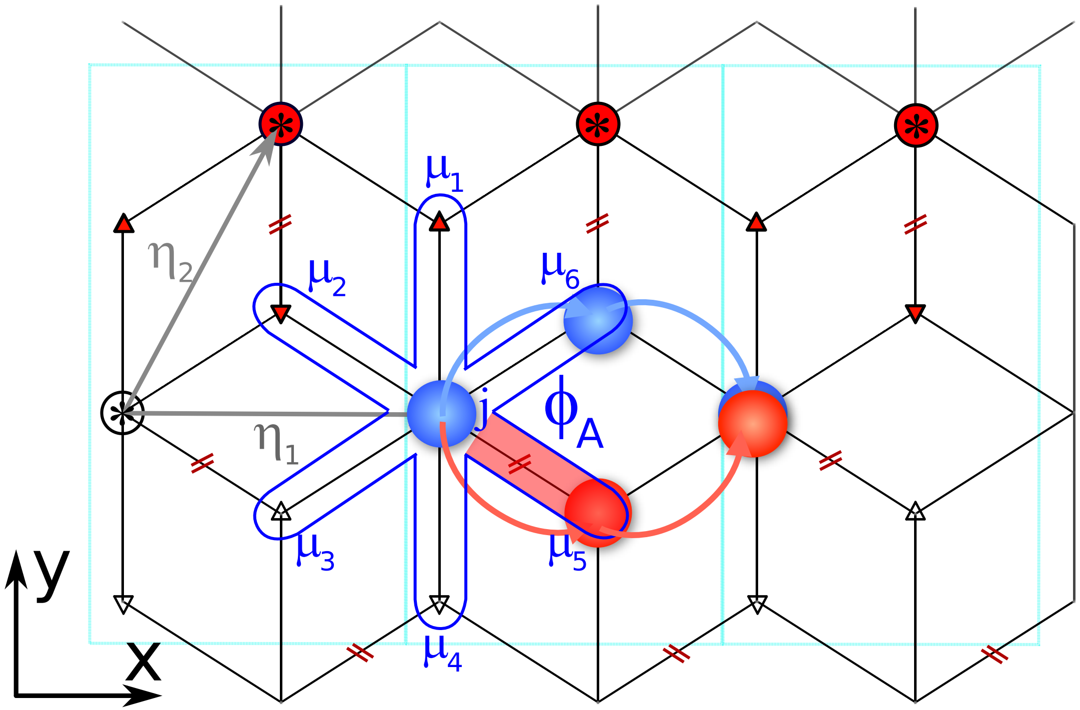


S1: standing wave to break mirror symmetry

P1...P6: propagating perpendicular to plane (P3 phase shifted)

L1, L2: propagating with in-plane momentum

Hamiltonian & Single particle wavefunctions



$$\mathcal{H} = -t \sum_{<j,\mu>} (\hat{a}^\dagger_\mu \hat{a}_j e^{iA_{\mu j}} + h.c.)$$

1. Choose a gauge $A_{\mu j}$

Note 1: can choose H real!
<u>Time-Reversal Symmetric</u>

Note 2: magnetic unit cell has 6 sites at flux density 1/2

2. Understand SP states

"Aharonov-Bohm cages"

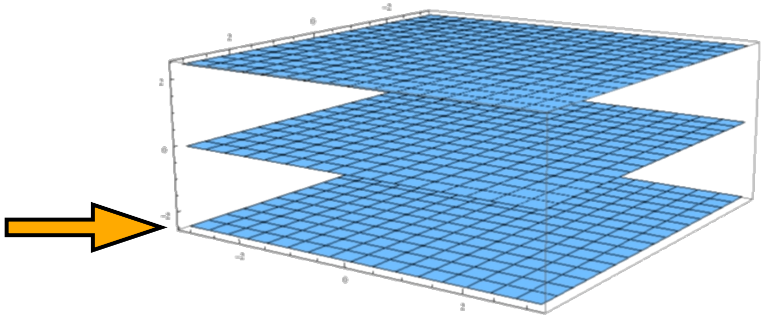
Vidal, Mosseri, Douçot, PRL (1998)

Next: add Interactions!
Assume onsite interactions
can have $U_6 \neq U_3$

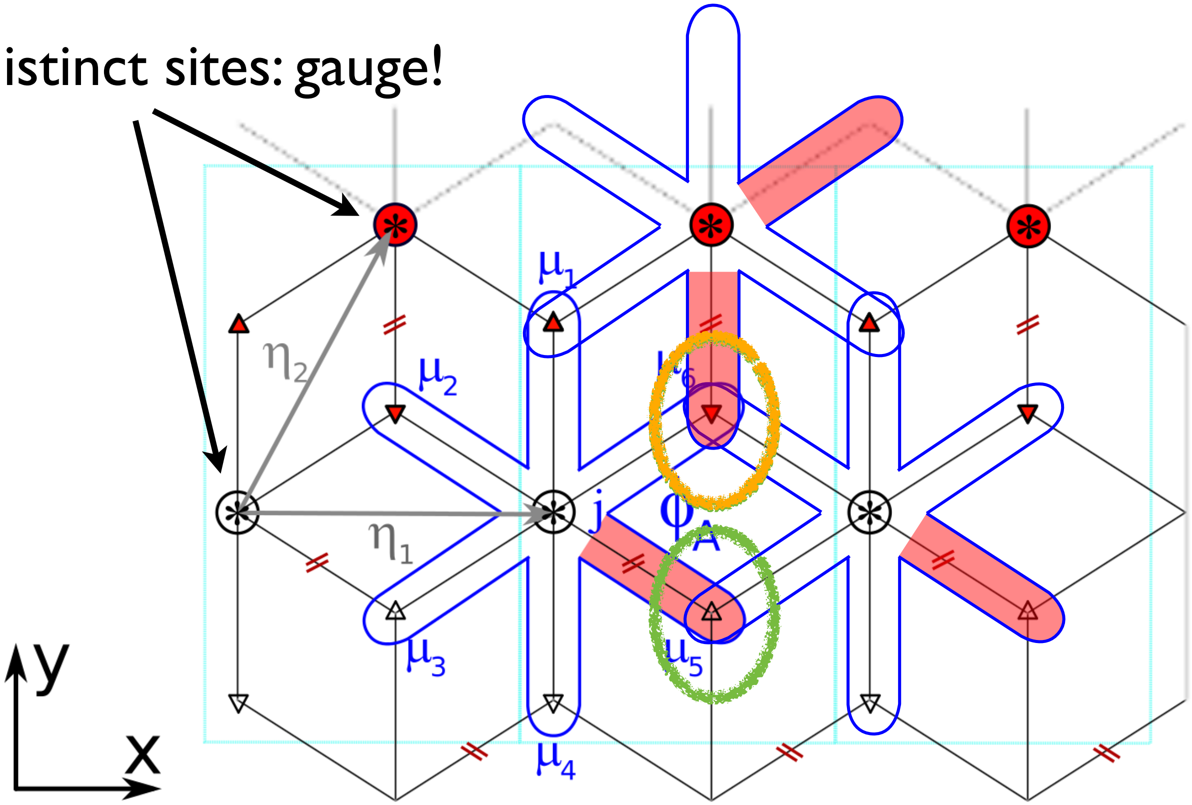
Effective dynamics and projection to lowest band

weak interactions: $nU/t \ll 1$

project to lowest band!



distinct sites: gauge!



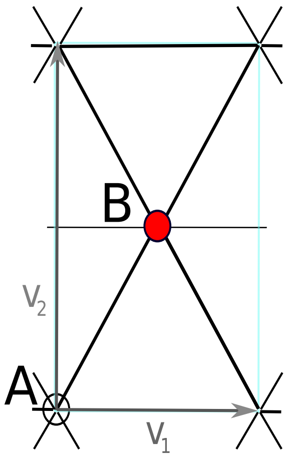
Matrix elements:

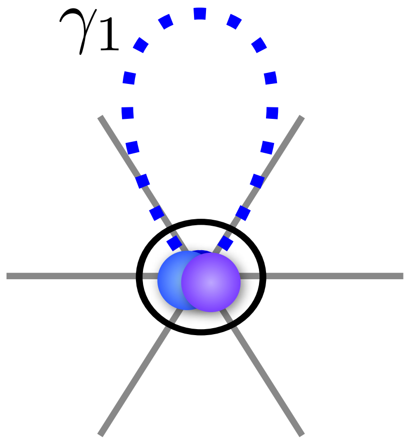
$$V_{ijkl} = U_3 \sum_\mu \phi_i^*(\mu)\phi_j^*(\mu)\phi_k(\mu)\phi_l(\mu) + U_6 \sum_q \phi_i^*(q)\phi_j^*(q)\phi_k(q)\phi_l(q)$$

i,j,k,l label orbitals in lowest band, localized around the sites of a the triangular lattice

Resulting model: "Activated Hopping"

- projected model lives on triangular lattice!





1. Onsite Interactions

$$\mathcal{H}_{\text{proj}} = \boxed{\gamma_1 \sum_i \hat{n}_i(\hat{n}_i - 1)} + \gamma_2 \sum_{\langle i,j\rangle} \left[\hat{n}_i\hat{n}_j + \hat{c}_i^{\dagger 2}\hat{c}_j^2 + \hat{c}_j^{\dagger 2}\hat{c}_i^2\right]$$

$$+\gamma_3 \sum_{\triangle(i,j,k)} \left[\sigma^{ij}_{kk}\hat{c}_i^\dagger\hat{c}_j^\dagger\hat{c}_k^2 + \sigma^{ik}_{jk}\hat{c}_i^\dagger\hat{c}_j\hat{n}_k + h.c.\right]$$

where: $\gamma_1 = \frac{1}{4}U_6 + \frac{1}{24}U_3,\ \gamma_2 = \frac{1}{72}U_3,\ \gamma_3 = \frac{1}{144}U_3$

Resulting model: "Activated Hopping"

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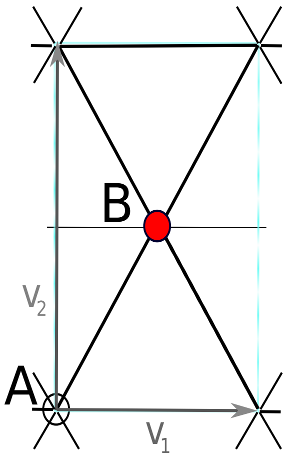

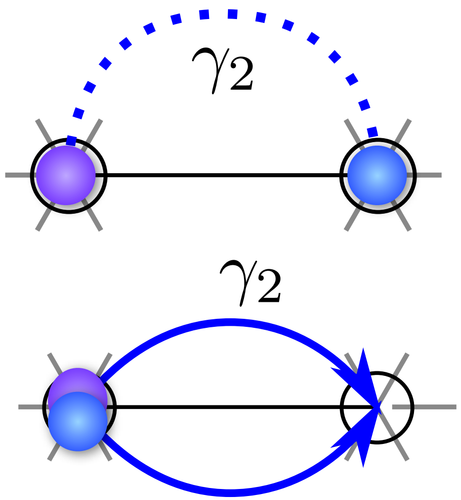

2a. Nearest Neighbor Interactions

2b. Correlated Pair Hopping

$$\mathcal{H}_{\text{proj}} = \gamma_1 \sum_i \hat{n}_i(\hat{n}_i - 1) + \gamma_2 \sum_{\langle i,j \rangle} \left[\hat{n}_i \hat{n}_j + \hat{c}_i^{\dagger 2} \hat{c}_j^2 + \hat{c}_j^{\dagger 2} \hat{c}_i^2 \right]$$

$$+ \gamma_3 \sum_{\triangle(i,j,k)} \left[\sigma_{kk}^{ij} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k^2 + \sigma_{jk}^{ik} \hat{c}_i^\dagger \hat{c}_j \hat{n}_k + h.c. \right]$$

where: $\gamma_1 = \frac{1}{4} U_6 + \frac{1}{24} U_3, \ \gamma_2 = \frac{1}{72} U_3, \ \gamma_3 = \frac{1}{144} U_3$

Resulting model: "Activated Hopping"

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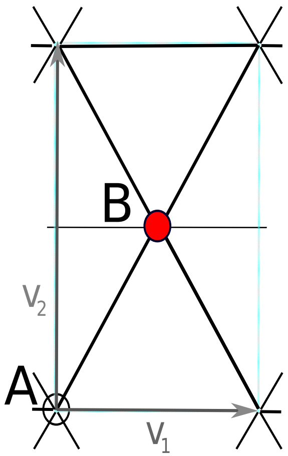
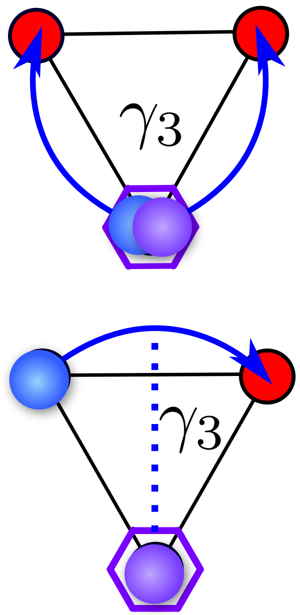
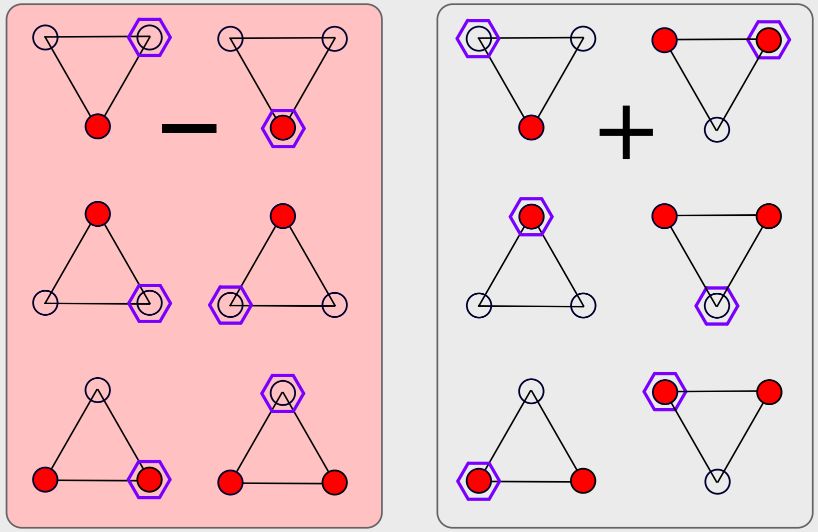




signs/phases for 3-orbital terms



$$\mathcal{H}_{\rm proj} = \gamma_1 \sum_i \hat{n}_i(\hat{n}_i - 1) + \gamma_2 \sum_{\langle i,j \rangle} \left[\hat{n}_i \hat{n}_j + \hat{c}_i^{\dagger 2} \hat{c}_j^2 + \hat{c}_j^{\dagger 2} \hat{c}_i^2 \right]$$

$$+ \gamma_3 \sum_{\triangle(i,j,k)} \left[\sigma^{ij}_{kk} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k^2 + \sigma^{ik}_{jk} \hat{c}_i^\dagger \hat{c}_j \hat{n}_k + h.c. \right]$$

where: $\gamma_1 = \frac{1}{4} U_6 + \frac{1}{24} U_3, \; \gamma_2 = \frac{1}{72} U_3, \; \gamma_3 = \frac{1}{144} U_3$

Resulting model: "Activated Hopping"

$$\mathcal{H}_{\text{proj}} = \gamma_1 \sum_i \hat{n}_i(\hat{n}_i - 1) + \gamma_2 \sum_{\langle i,j \rangle} \left[\hat{n}_i \hat{n}_j + \hat{c}_i^{\dagger 2} \hat{c}_j^2 + \hat{c}_j^{\dagger 2} \hat{c}_i^2 \right]$$

$$+ \gamma_3 \sum_{\triangle(i,j,k)} \left[\sigma_{kk}^{ij} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k^2 + \sigma_{jk}^{ik} \hat{c}_i^\dagger \hat{c}_j \hat{n}_k + h.c. \right]$$

where: $\gamma_1 = \frac{1}{4} U_6 + \frac{1}{24} U_3,\ \gamma_2 = \frac{1}{72} U_3,\ \gamma_3 = \frac{1}{144} U_3$

- Projected model has finite range interactions
- Time reversal symmetric
- All interactions are of density-density type
- Can tune ratio of onsite terms γ_1 to activated hopping terms $\gamma_{2,3}$ via the free parameter

$$u = U_6 / U_3$$

1. Gross-Pitaevski Mean Field Theory

$$|\Psi\rangle = \exp\left\{\sum_j \alpha_j \hat{c}_j^\dagger\right\}|\text{vac.}\rangle$$

2. Exact Numerical Diagonalization

$$\mathcal{H}|\Psi\rangle = E|\Psi\rangle$$

$$|\Psi\rangle = \sum_\alpha \kappa_\alpha \prod_{i=1}^{N_{\text{orb}}} (\hat{c}_i^\dagger)^{n_i(\alpha)}|\text{vac.}\rangle$$

- Note: i,j label orbitals in lowest band

Gross-Pitaevski Mean-Field Theory I

- Regime of validity for Ansatz $|\Psi\rangle = \exp\left\{\sum_j \alpha_j \hat{c}_j^\dagger\right\}|\text{vac.}\rangle$
  - Large density $n \gg 1$
  - Remain in projected model $nU \ll t$
  - Grand canonical ensemble: introduce chemical potential μ
- Minimize expectation value of energy given μ

$$\langle H\rangle = \sum_{ijkl} V_{ijkl}\alpha_i^*\alpha_j^*\alpha_k\alpha_l - \mu\sum_j |\alpha_j|^2$$

  - Note: i,j,k,l label orbitals in lowest band

Gross-Pitaevski Mean-Field Theory II

- Groundstates are vortex lattices when expanded on full dice lattice (all sites)

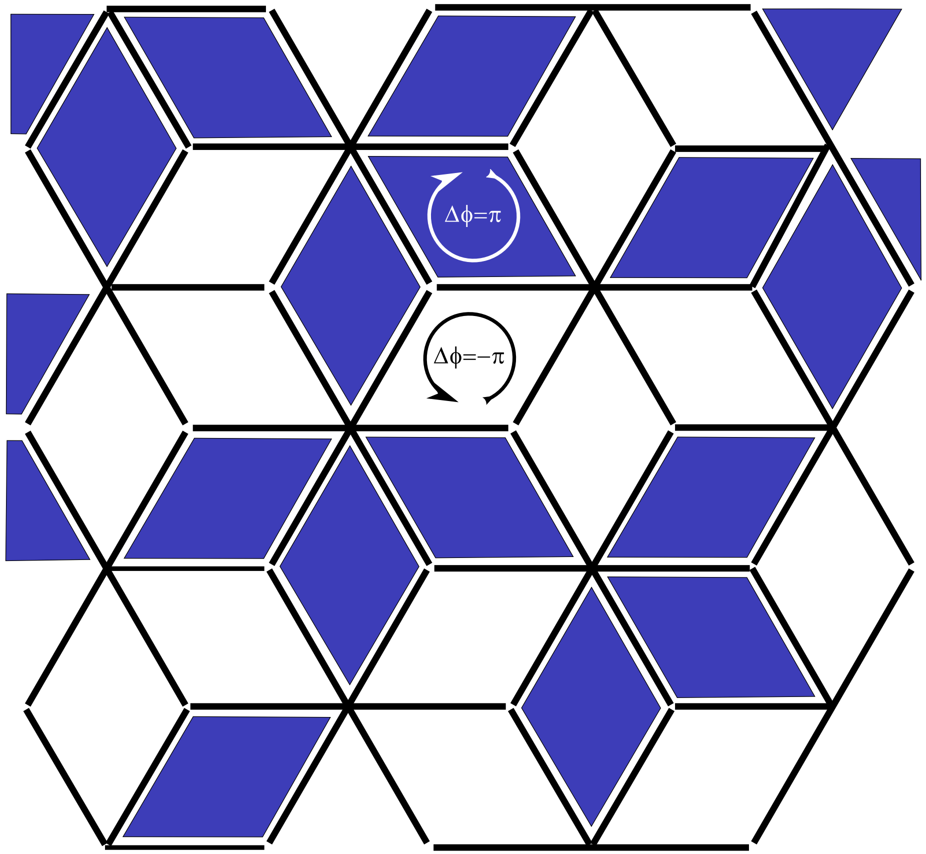
- density: distinct values on 6-fold hubs and 3-fold rims

$$\text{for } u = 1\text{: } n_6 = 2n_3$$

- phases correspond to groundstate solutions of fully-frustrated xy-model on the dice lattice

S. E. Korshunov, Phys. Rev. B, 63 (2001)

$n \gg 1$



GM & N.R. Cooper, PRL 108, 043506 (2012)

- π-Flux vortex lattices with no more than three neighboring plaquettes with same sign of vorticity

Gross-Pitaevski Mean-Field Theory II

- More examples of groundstate vortex patterns

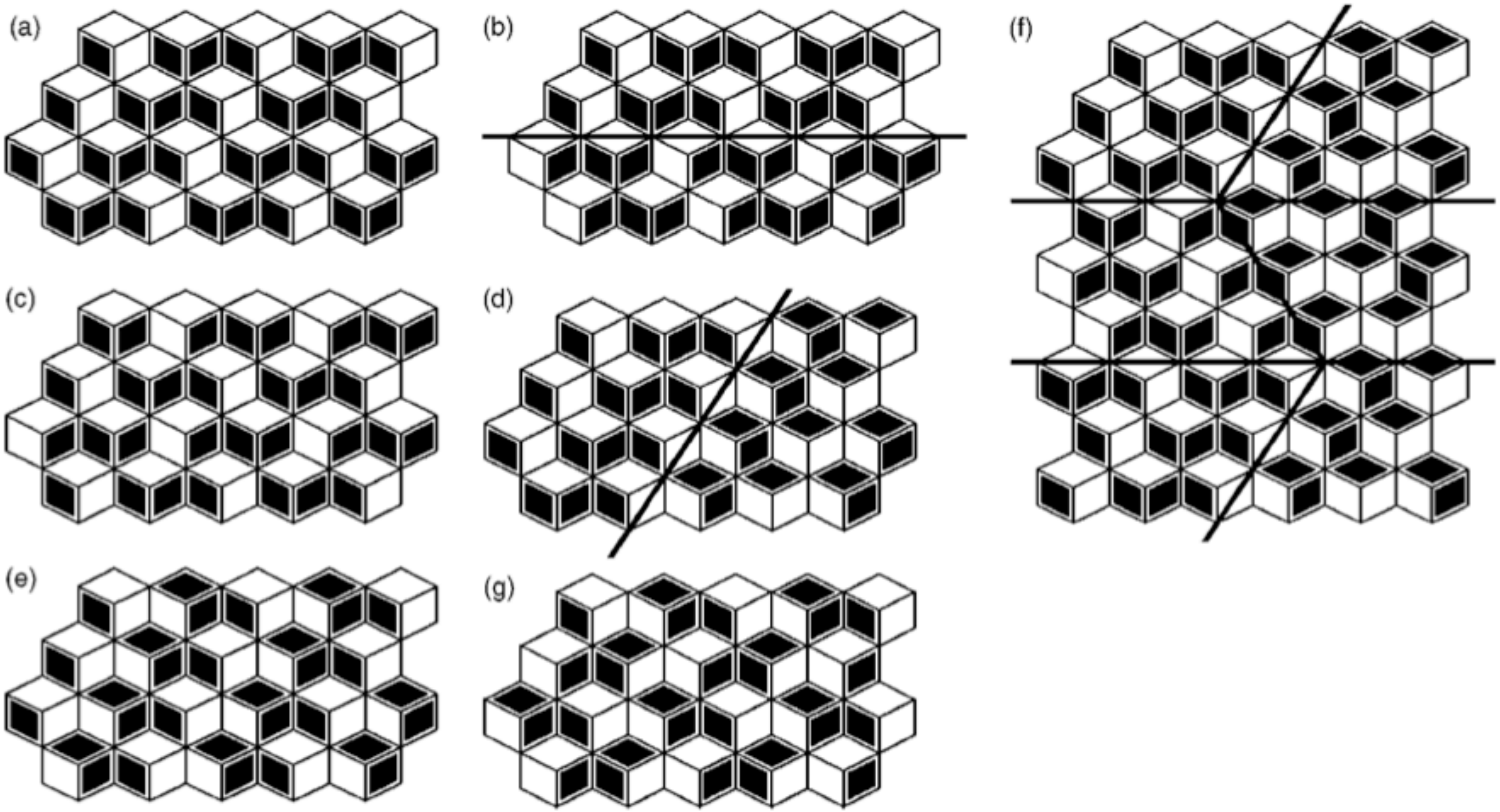
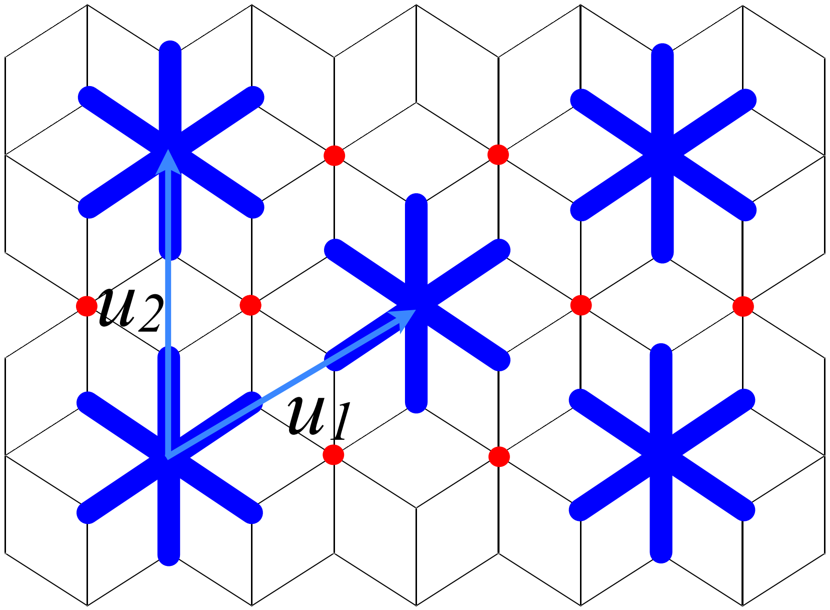


illustration credits: S. E. Korshunov, Phys. Rev. B, 71 (2005)

Low density limit

$\mathcal{H}_{proj}$ does not induce any dynamics for particles which are not nearest neighbors



- Densest packing of particles in lowest band yields 3x deg.

crystalline groundstate at band filling $\nu = 1/3$

$$|\Psi_c\rangle = \prod_{n,m} \hat{c}^\dagger[\vec{r}_t + n\vec{u}_1 + m\vec{u}_2]|\text{vac.}\rangle$$

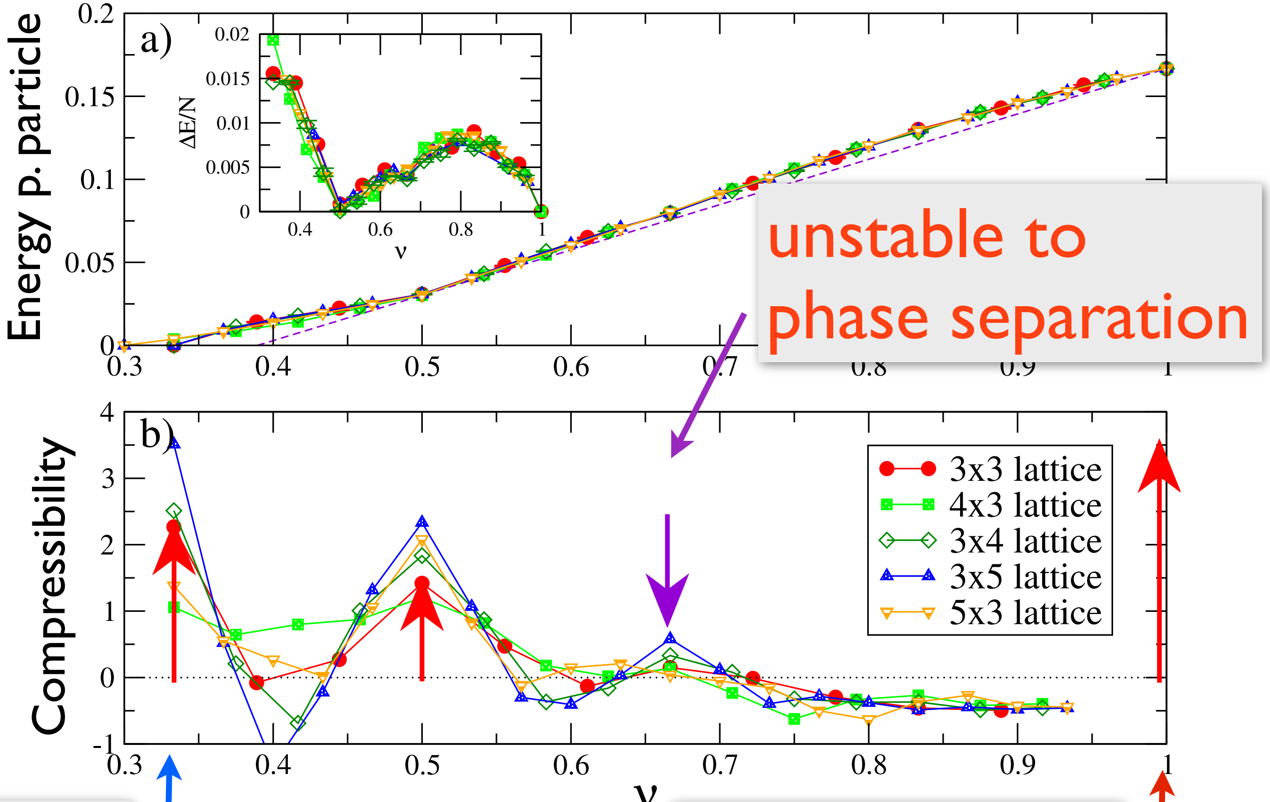
Correlated regimes at intermediate particle density?

- How do fluctuations affect the highly degenerate groundstate?
- How do fluctuations disorder the crystalline phases at small n?

ED, using limit of large $u \gg 1$

- analyse different finite size lattices with periodic boundary conditions
- vary band filling $\nu = 3n$

unidentified phase at $\nu = 1/2$

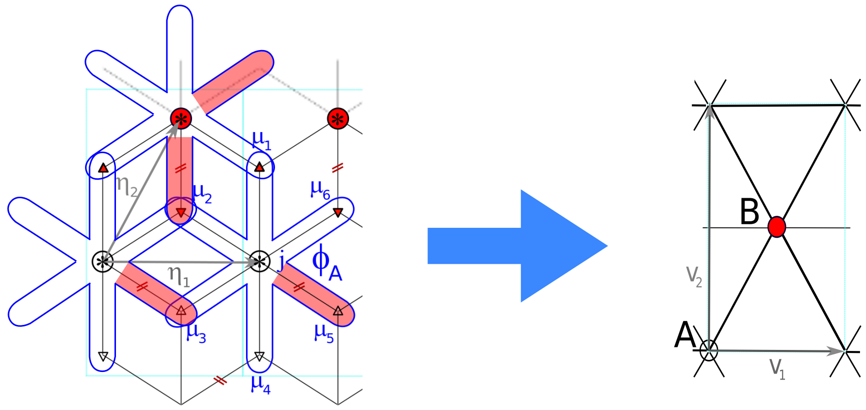


crystal

Mott Insulator

Characteristics of the phase at filling 1/2

‣ Reminder: projection yields triangular lattice

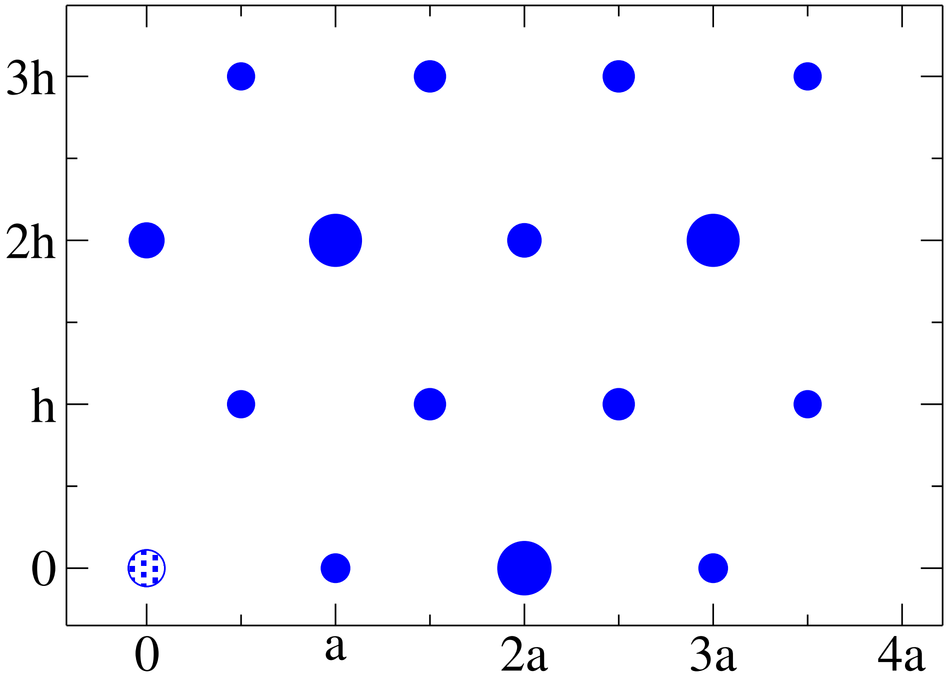


‣ Projection reduces system size to 1/3 of original number of sites.

‣ Can achieve system size of 16 particles on 32 sites for (hard-core) bosons at half filling, corresponding to 96 sites of the dice lattice!

Characteristics of the phase at filling 1/2

- Particle-Particle Correlations (exact groundstate, 4x2 unit cells)



- symbol size ~

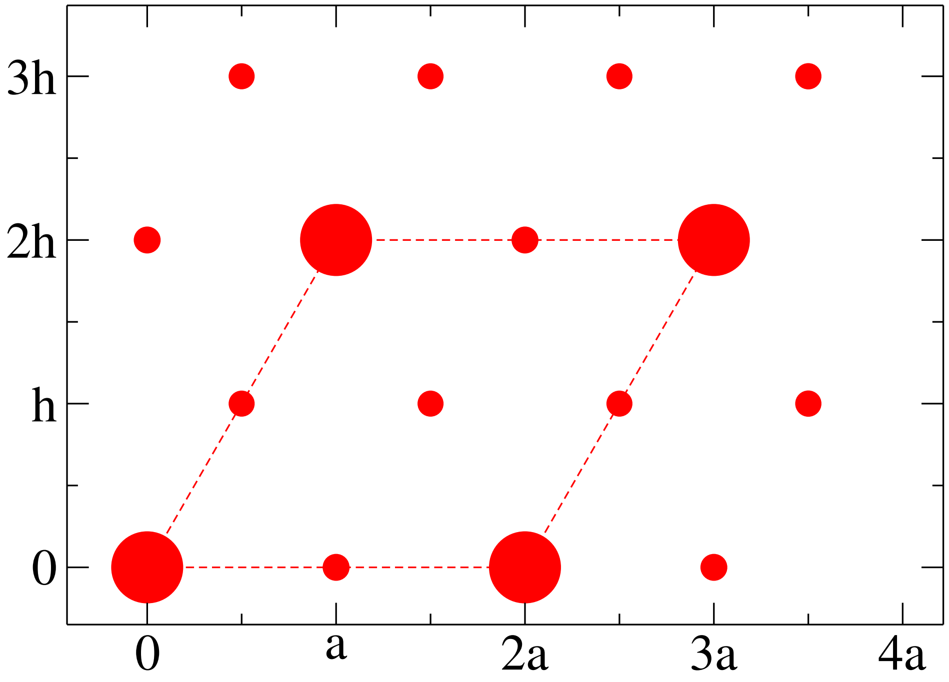
$$\langle \Psi | \hat{n}(\vec{r}) \hat{n}(\vec{0}) | \Psi \rangle$$

- correlations indicate crystalline long-range order with basis

$$(2\vec{\eta}_1, 2\vec{\eta}_2)$$

Characteristics of the phase at filling 1/2

- Break translational symmetry explicitly, taking superposition of four lowest-lying eigenstates $|S\rangle$



- symbol size ~

$$\langle S|\hat{n}(\vec{r})|S\rangle$$

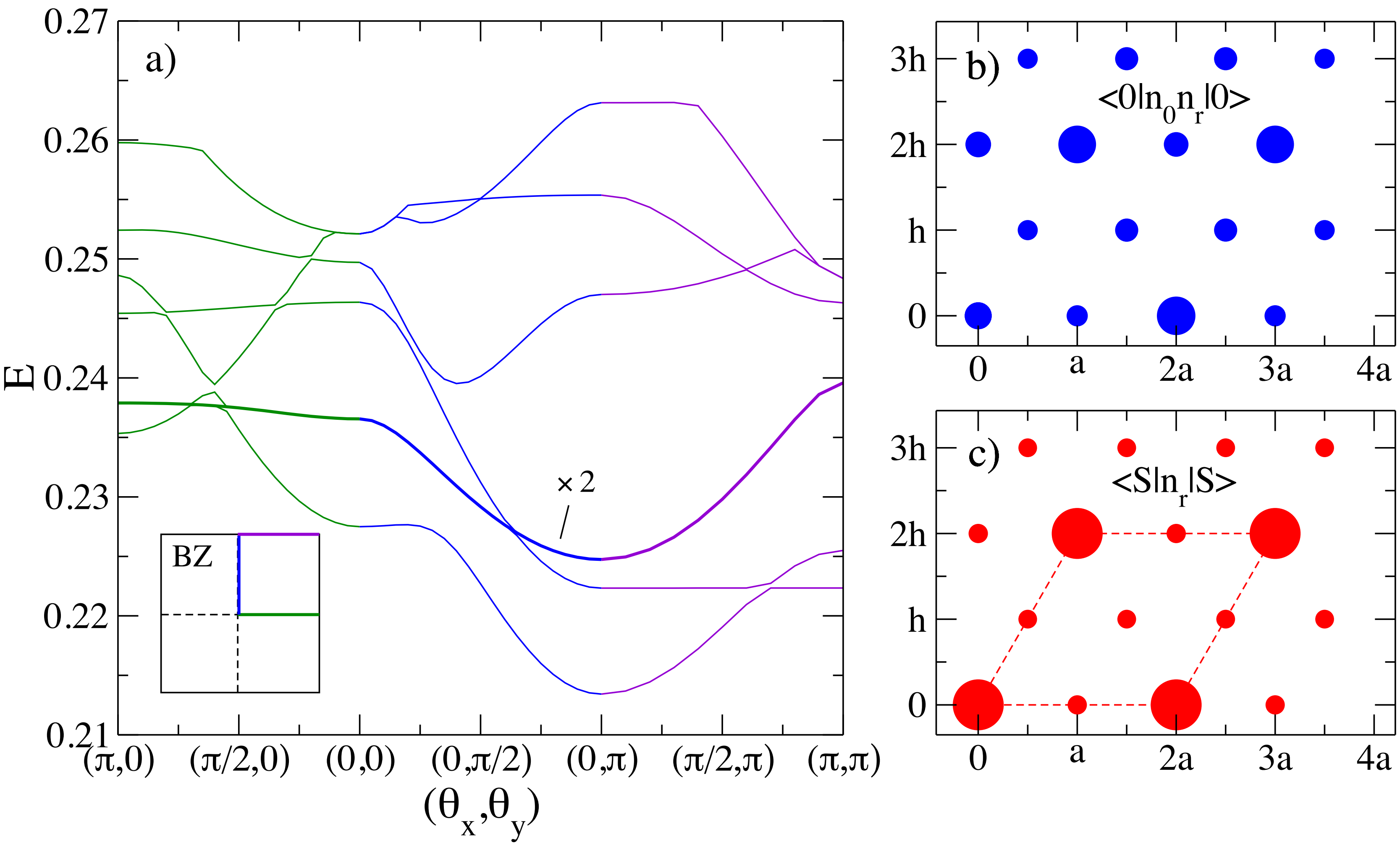
$$|S\rangle = \sum_{i=1}^{4} c_i |\Psi_i\rangle$$

- coefficients: see next slide

- confirms crystalline order anticipated from correlations

$$(2\vec{\eta}_1, 2\vec{\eta}_2)$$

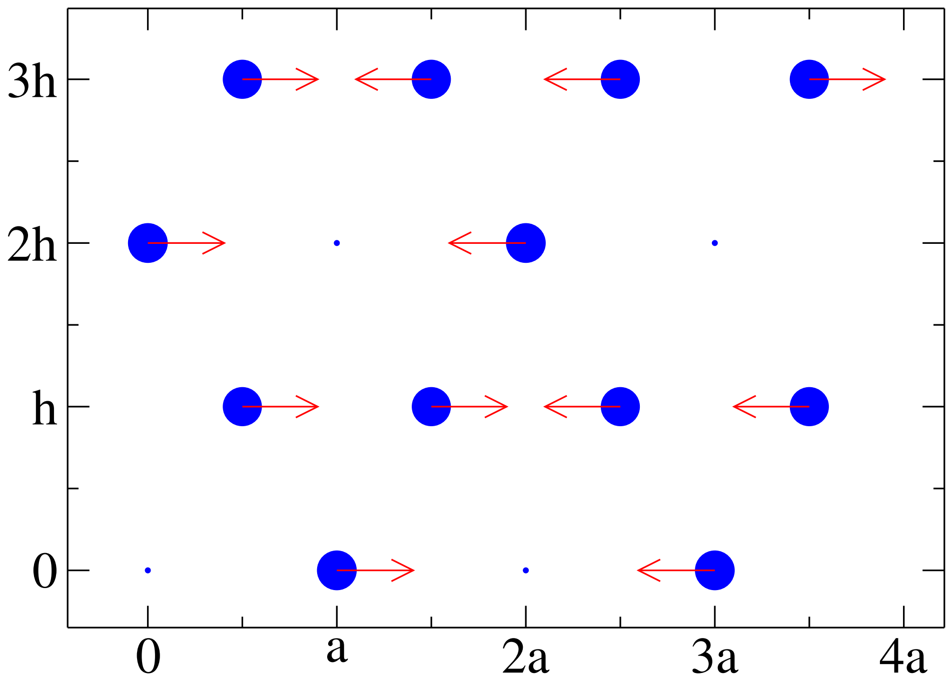
Spectrum under twist of boundary conditions



- finite spin stiffness is indicative of a superfluid component

Condensate fraction at filling 1/2?

- Construction of state $|S\rangle$: maximize largest eigenvalue of single-particle density matrix $\rho_{ij} = \langle \hat{c}_i^\dagger \hat{c}_j \rangle$



- blue dot size ~ particle fluctuations

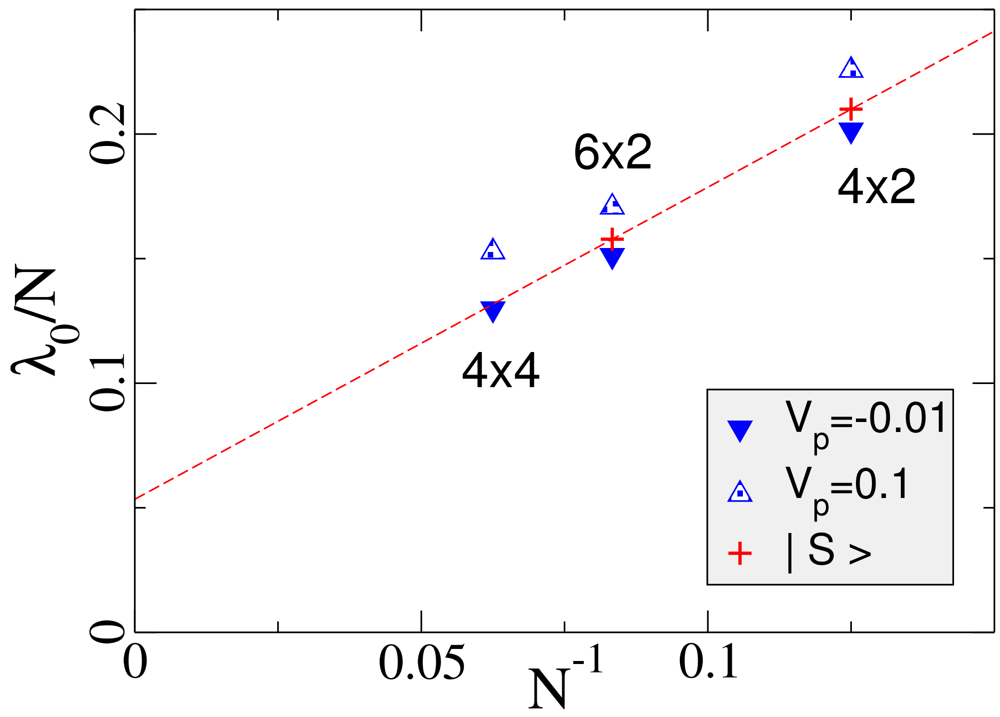
$$\langle S|\hat{n}^2(\vec{r})|S\rangle - \bar{n}^2$$

- red arrows: magnitude and phase of largest eigenvector of ρ = condensate wavefunction

- Suggests simultaneous presence of both crystalline and superfluid fraction!

Finite Condensate fraction in thermodynamic limit?

- Examine finite size scaling of condensate fraction



- confirms the presence of a supersolid ordered phase in the half filled band! Experimental detection: *in-situ / expansion*

Conclusions I

- Dice lattice is an attractive model to study many-body physics in flat, time-reversal symmetric bands

- The model has a rich phase diagram, including:
  - Crystals, Mott Insulators
  - Supersolids, intricate degenerate vortex lattice phases

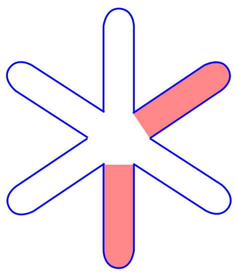
- Activated hopping processes emerge as a generic feature of topologically trivial flat band models, and drive the formation of this complex phase diagram

GM & N.R. Cooper, Physical Review Letters 108, 043506 (2012)

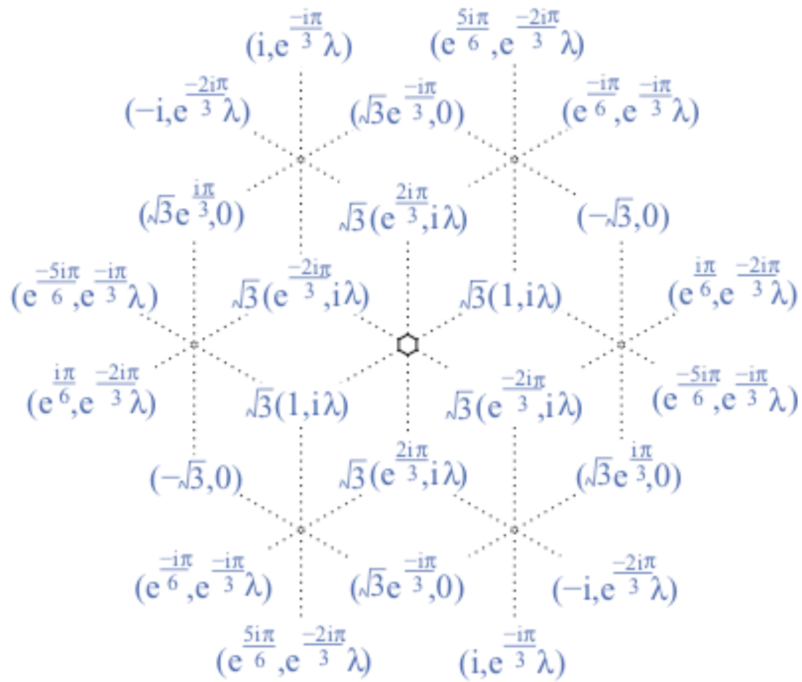
Differences between TRI and Chern bands

- Maximally localized wavefunctions in bands with Chern numbers $C>0$ are more extended

$C = 0$



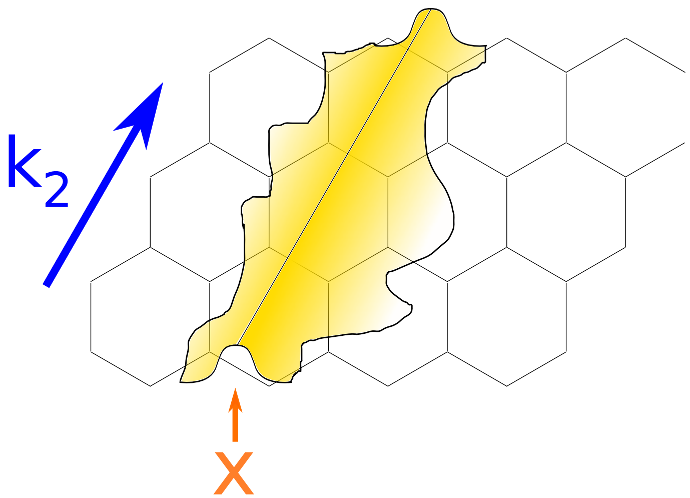
$C = 2$

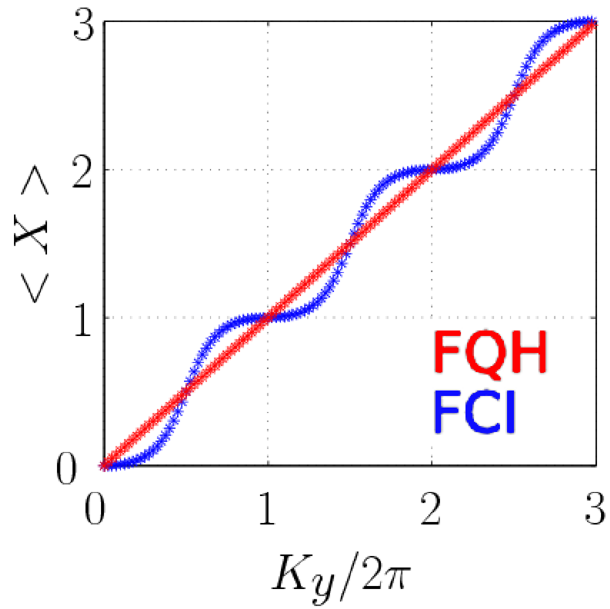


graphics: Wang & Ran, PRB (2011)

Hybrid Wannier functions in Chern bands

- To visualise, think about localising wavefunctions in one direction, first





$$|W(x,k_y)\rangle = \sum_{k_x} f_{k_x}^{(x,k_y)} |k_x, k_y\rangle$$

- Increase in position for $k_y \rightarrow k_y + 2\pi$
  $=$ Chern-number C

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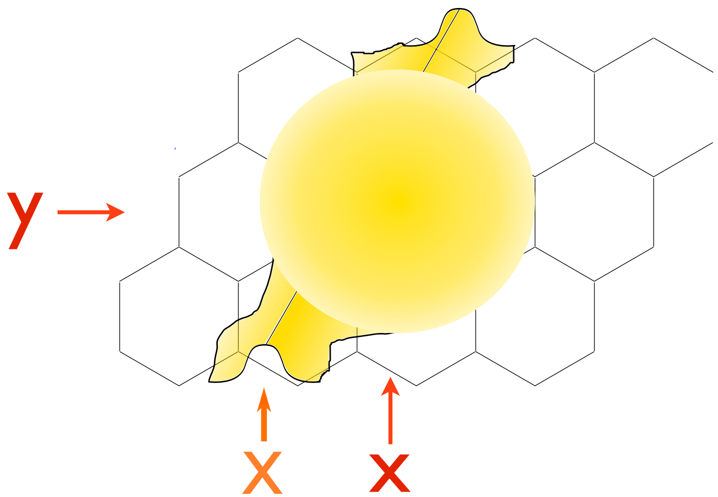
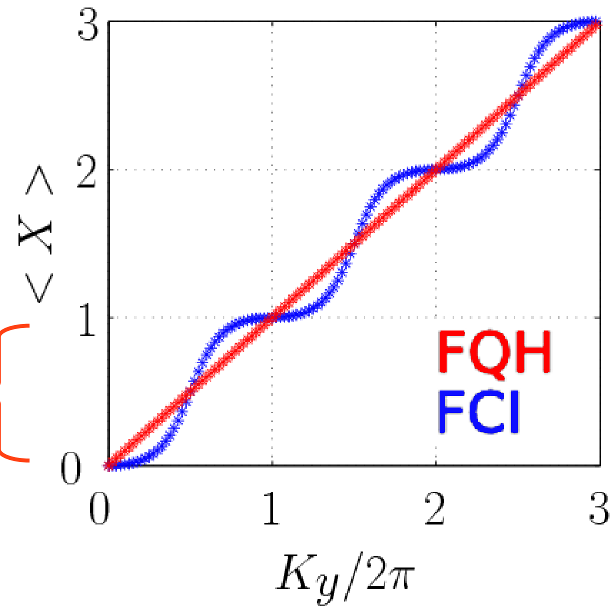
- Fully localized state mixes different momenta k_y and thus positions $<x>$



- mixed positions contribute



$$|\tilde{W}(x,y)\rangle = \sum_{k_y} g_{k_y} |W(x,k_y)\rangle$$

- Increase in position for $k_y \rightarrow k_y + 2\pi$ $=$ Chern-number C

- fully localized Wannier states have an extent of ca. *(C+1)* unit cells

Advert: Analytic Continuation between FQHE and FCI

- Can use single particle Wannier states to construct an analytic continuation between incompressible quantum liquids in C=1 Chern bands (fractional Chern insulators) and the fractional QHE

‣ for a numerical study of fractional Chern insulators in the hybrid Wannier basis, see Th. Scaffidi & GM, PRL 109, 246805 (2012).