Creating novel quantum phases by artificial magnetic fields

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Outline

- A brief introduction to the quantum Hall effect
- Topological aspects of Landau levels and how to simulate magnetic fields

- Adiabatic connection of fractionalized phases in topologically non-trivial Chern bands and FQH states

- Novel types of quantum liquids bands Chern-# $C > 1$
  - composite fermion approach for bosons in flux lattices
  - flat band projection in the Hofstadter butterfly

Quantum Hall Effect: Phenomenology

- a macroscopic quantum phenomenon observed in magnetoresistance measurements

Where?
- in semiconductor hetero-structures with clean two-dimensional electron gases
- at low temperatures (~0.1K) and in strong magnetic fields

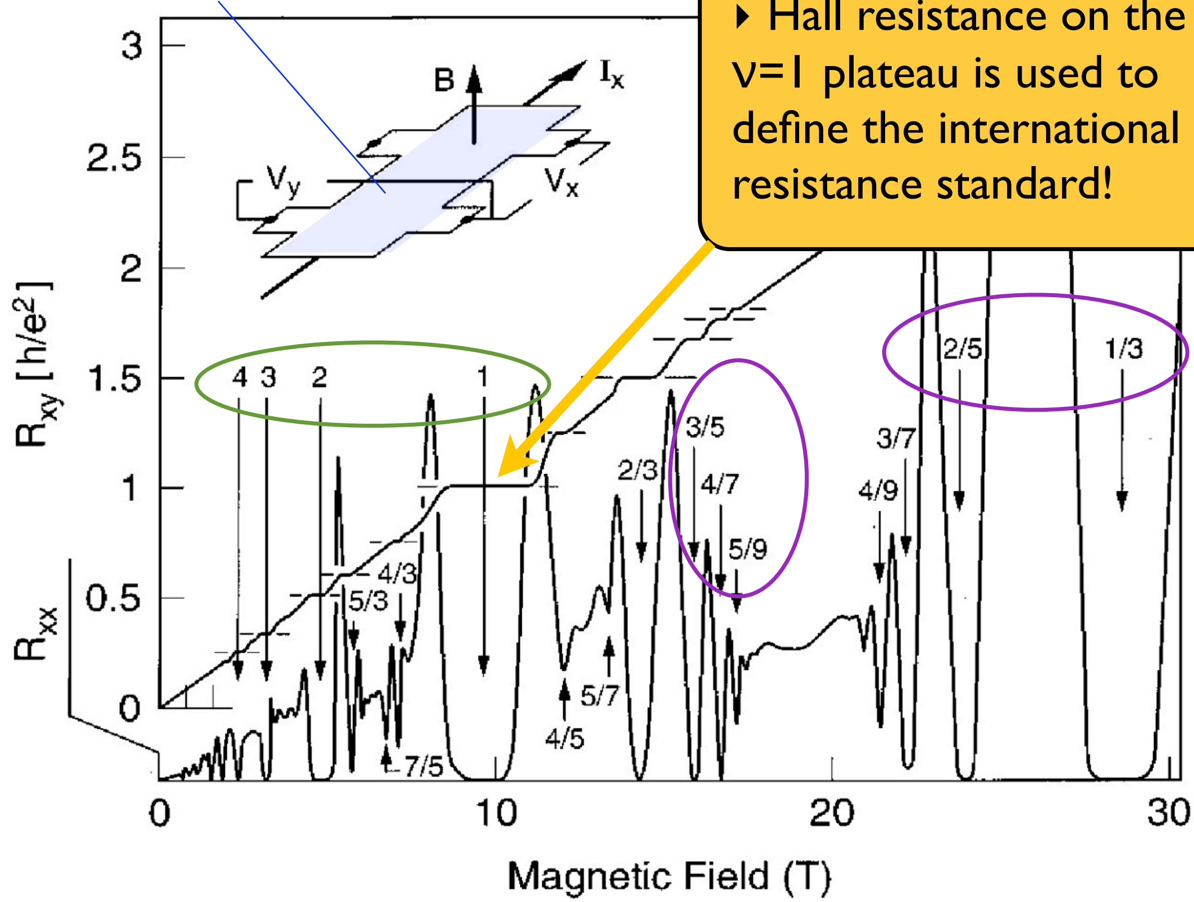
$$k_B T \ll \hbar\omega_c = \hbar e B / m_e$$

What?
- plateaus in Hall conductance

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

- simultaneously: (near) zero longitudinal resistance

2D electron gas



- Hall resistance on the ν=1 plateau is used to define the international resistance standard!

- Quantum number ν, observed to take integer or simple fractional values

Integer Quantum Hall Effect

$$\mathcal{H} = \frac{(\vec{p} + e\vec{A})^2}{2m}$$

$$\vec{A} = Bx\vec{e}_y$$

- single-particle eigenstates (=bands) in a homogeneous magnetic field: degenerate Landau levels with spacing $\hbar\omega_c$

$$\omega_c = eB/m_e \text{ cyclotron frequency}$$

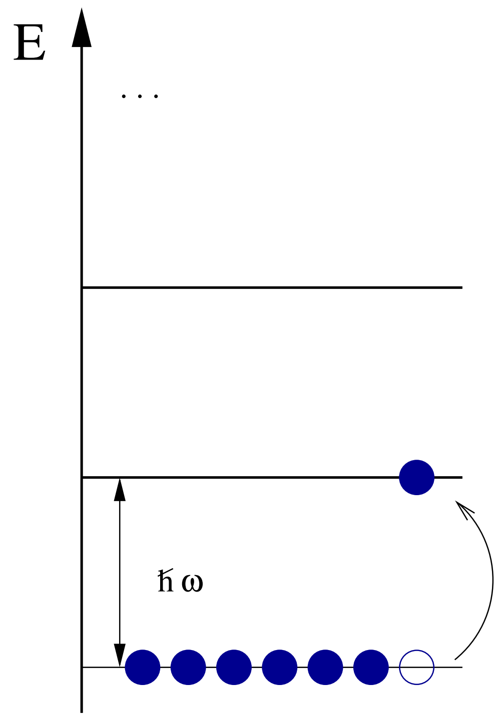
- degeneracy per surface area: $d_{LL} = eB/h$

- fill a number of bands = integer filling factor $\nu = n/d_{LL}$

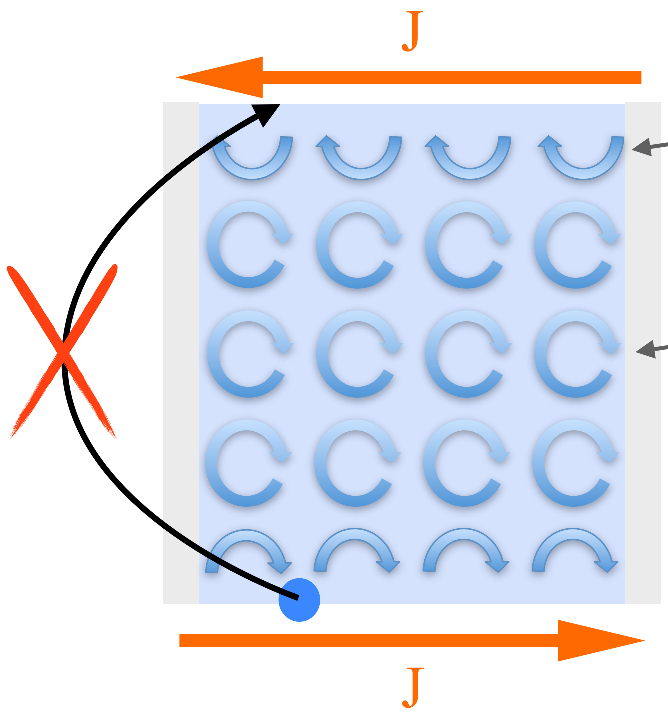
⇒ large gap Δ for single particle excitations:

naively, we should have a band insulator

- There must be something special about Landau-levels!



Semiclassical picture: skipping orbits



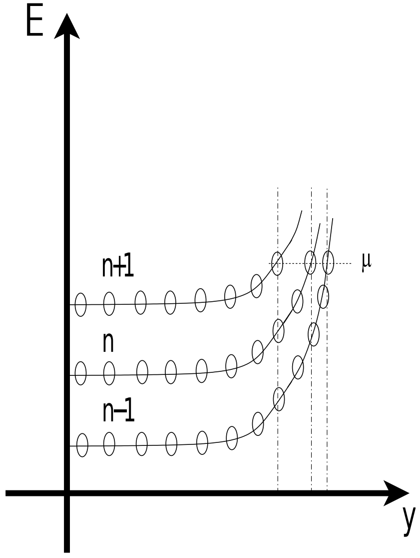
at edge of sample, 'skipping orbits' contribute a uni-directional current

cyclotron motion produces no net current in bulk of sample

picture for quantum transport: absence of backscattering
⇒ dissipationless current

⇒ no voltage drop along lead!

several LLs filled:



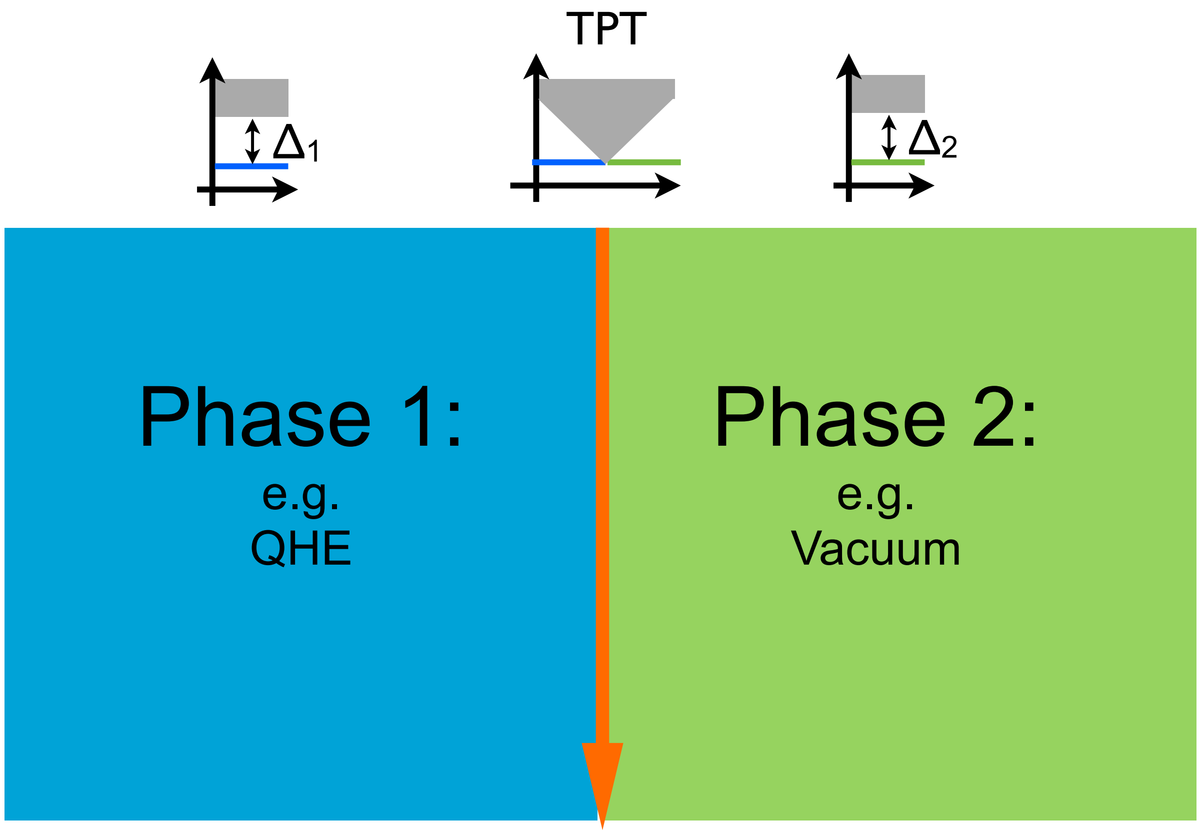
low-energy or 'gapless' excitations present near boundary

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

Edge States & Topological Order

Quantum Hall plateaus have a property called *topological order*

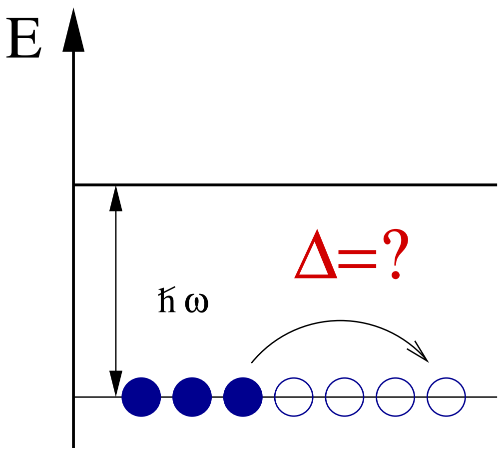
Edge states must occur where the topological order changes in space (gap must close)

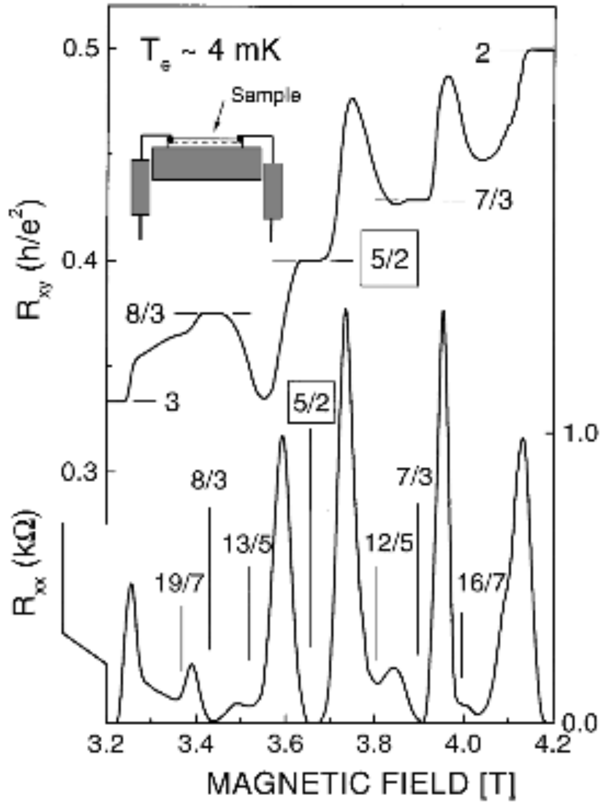


Can formulate *topological invariants* to characterise topological order: will see example, later

Fractional Quantum Hall Effect (FQHE)

- plateaus seen also for non-integer ν
- not filled bands - but similar phenomenology as integer filling:



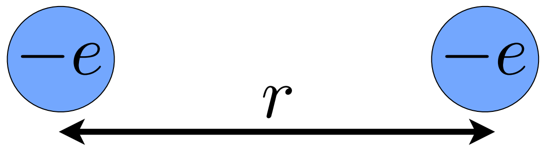


- nature of interactions determines how the system behaves:

$$\mathcal{H} = \sum_i \frac{1}{2m}(\vec{p}_i - e\vec{A}_i)^2 + \sum_{i<j} V(|\vec{r}_i - \vec{r}_j|)$$

within Landau-level: Kinetic Energy=constant

interactions determine quantum state



$$V(\mathbf{r}) = \frac{e^2}{4\pi\epsilon_0 |r|}$$

⇒ FQHE is an inherently many-body phenomenon

- each Hall plateau represents a kind of topological order

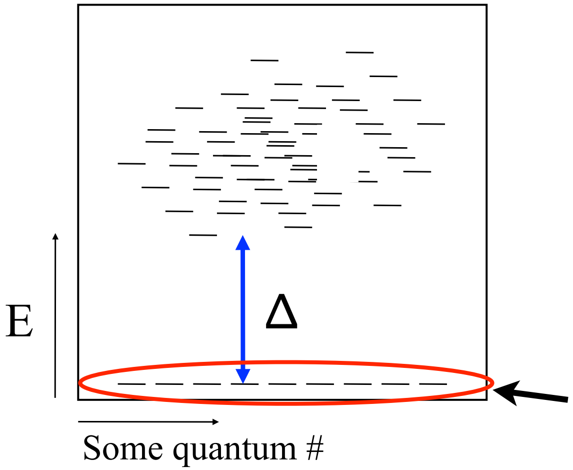
Why is the fractional quantum Hall effect important?

source of very unusual physics, for example:

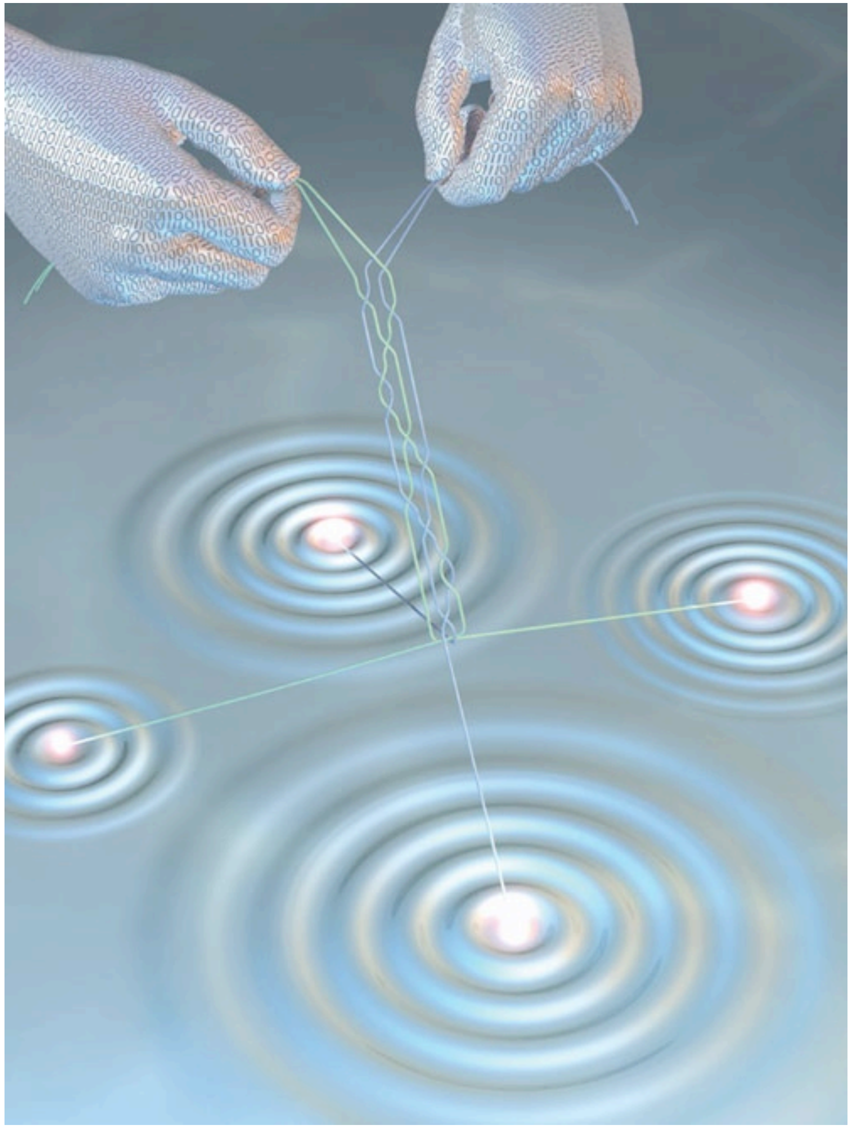
- quasi-particles with fractional electronic charge

$$e.g.,\ q = e/3$$

- manipulations of quasiparticles could provide the basis for a quantum computer that is protected from errors!

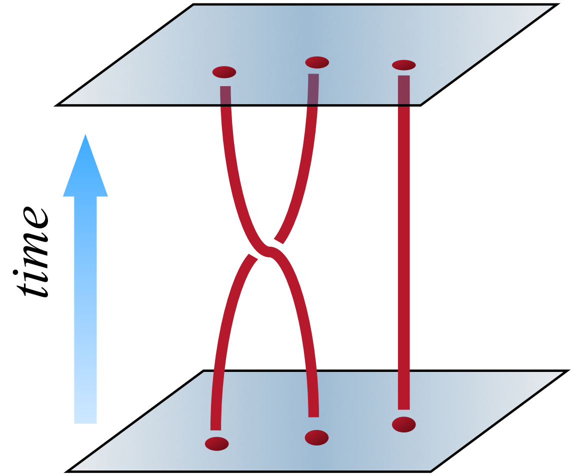


topological degeneracy of low-lying states



- quantum operations by braiding quasiparticles

Fractional statistics - Anyons and Non-Abelions

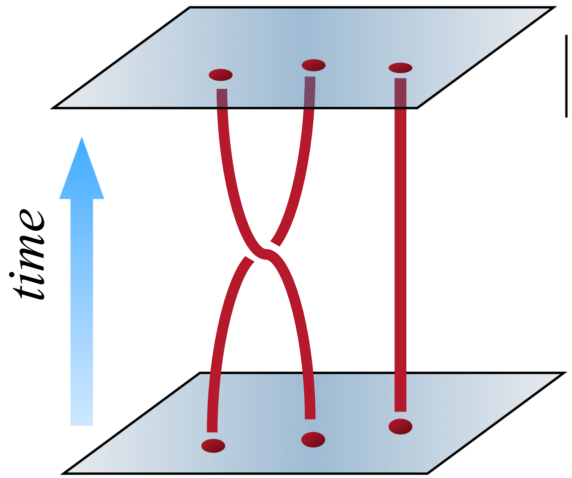


$$|\psi_f\rangle = e^{i\phi} |\psi_i\rangle$$

almost 'any' phase

$|\psi_i\rangle$

Abelian Anyons



$$|\psi_f\rangle = \begin{bmatrix} \tilde{\alpha} \\ \tilde{\beta} \end{bmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

Non-Abelian Anyons

non-Abelian representation of the braid group

$$|\psi_i\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

orthogonal degenerate states

Quantum Hall effect without magnetic fields

The fractional quantum Hall effect is observed under extreme conditions

- strong magnetic fields of several Tesla
- very low temperatures
- clean / high mobility semiconductor samples

Opportunities for creating novel types of quantum Hall systems

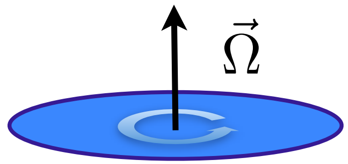
1. Cold Atomic Gases
   - both bosons and fermions
   - highly tuneable: density, interactions, tunnelling strengths, (effective) mass, ...
   - different types of experimental probes: local density, velocity distribution, correlations

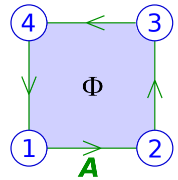
2. Novel classes of materials
   - strained graphene
   - materials with strong spin orbit coupling, such as topological insulators

Strategies for simulating magnetic fields

- Simulate a physical effect that a magnetic field B exerts particle of charge q

Signature		Simulated by
Lorentz Force	$F_L = q\,\vec{v} \times \vec{B}$	Coriolis Force in Rotating System
Aharonov-Bohm Effect	$\Psi \propto \exp\left\{ i\frac{q}{\hbar}\int \vec{A}\cdot d\vec{\ell}\right\}$	Complex Hopping Amplitudes A in Optical Lattices
Berry Curvature of Landau levels		?



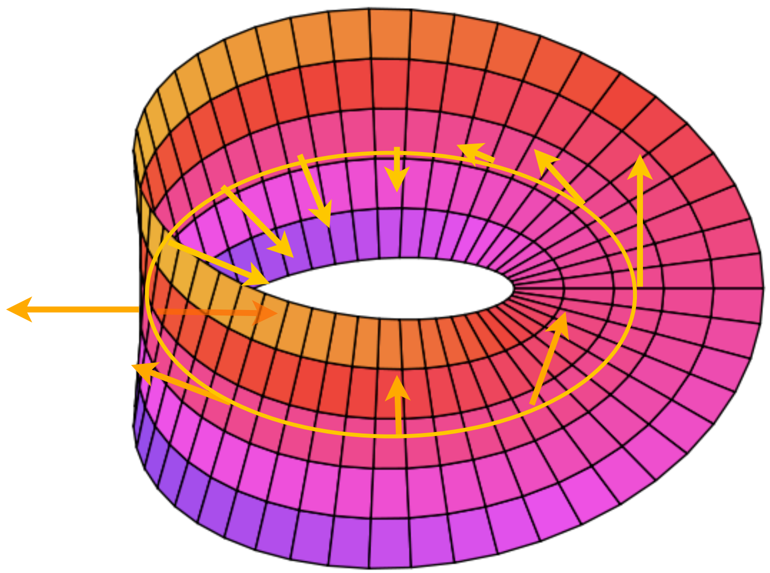


$$\sum_{\square} A_{\alpha\beta} = 2\pi n_\phi$$

Landau-levels as a topological band-structure

- Can we see in which way Landau-levels are special, just by looking at the wavefunctions?

Start with an analogy:



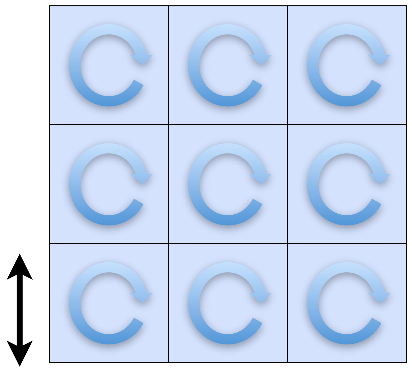
Recipe for calculating the twist in this Möbius band:

- choose a closed path around the surface
- construct normal vector to the surface at points along the curve
- add up the twist angle while moving along this contour

Calculating the 'twist' in wave functions I

1. How to think of the surface / manifold on which Landau level wavefunctions φ live?

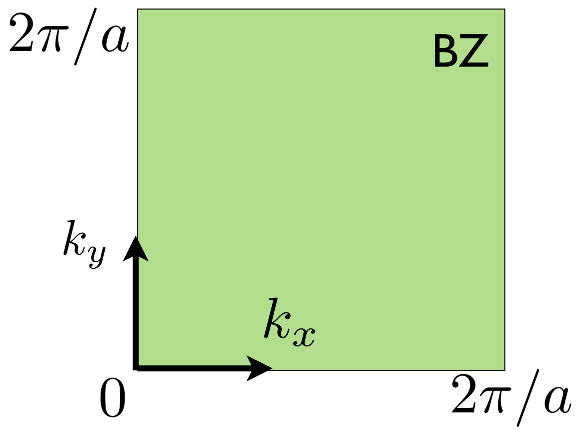
choose to make wavefunctions periodic (by superposition):



$$\varphi(\vec{r}+\vec{a}) \propto \varphi(\vec{r})$$

periodic repetitions of a unit cell (UC)

'crystal momentum' ***k*** parametrizes states in Brillouin zone (BZ)



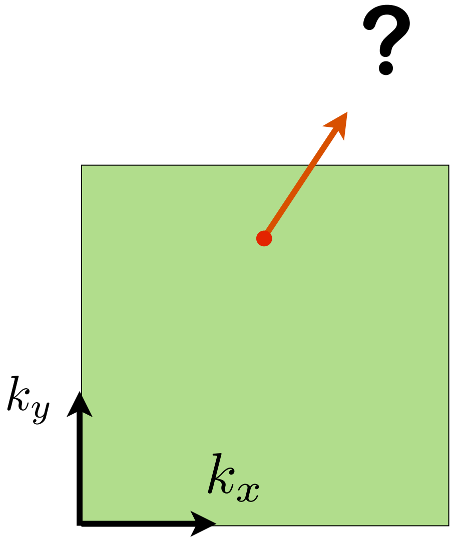
$$\varphi_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r})e^{i\vec{k}\vec{r}}$$ (Bloch's theorem)

notation for strictly periodic part

$$u_{\vec{k}}(\vec{r}+\vec{a}) = u_{\vec{k}}(\vec{r})$$

Calculating the 'twist' in wave functions II

2. What is the equivalent of the normal vector?



The wavefunction itself is a vector!

lives in a function space with a norm = Hilbert space

$$\varphi_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r}) e^{i\vec{k}\vec{r}}$$

having chosen a point ***k***, only u provides non-trivial information and use the ket representation

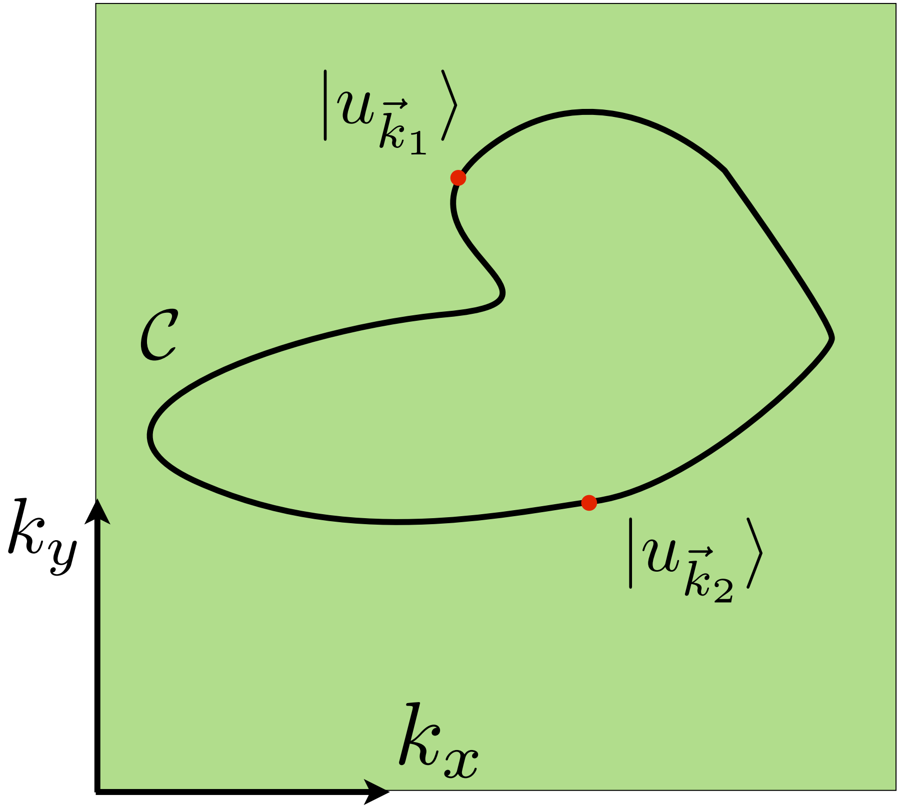
$$|u_{\vec{k}}\rangle$$

u's are eigenstates of a transformed Hamiltonian:

$$\tilde{\mathcal{H}} = e^{-i\vec{k}\vec{r}} \mathcal{H} e^{i\vec{k}\vec{r}} \quad \Rightarrow \quad \tilde{\mathcal{H}} |u_{\vec{k}}\rangle = \epsilon_k |u_{\vec{k}}\rangle$$

Calculating the 'twist' in wave functions III

3. What is the equivalent of the twist angle?



direction of vector fixed at any point ...

$$|u_{\vec{k}}\rangle$$

... up to an overall phase

$$e^{i\gamma}|u_{\vec{k}}\rangle$$

$\Rightarrow$ need to keep track of the phase!

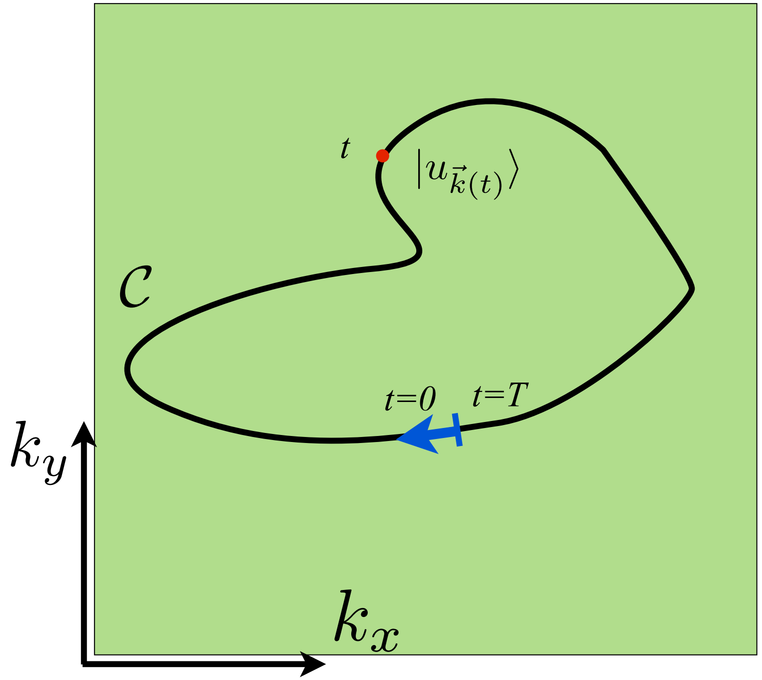
- use scalar product to compare vectors

Calculating the Berry phase

Michael Berry (1984)

Calculate how wavefunction evolves while moving adiabatically through curve C : $\mathbf{k}(t)$, $t=0...T$

Local basis $\tilde{\mathcal{H}}|u_{\vec{k}}\rangle = \epsilon_{\vec{k}}|u_{\vec{k}}\rangle$



Phase evolution has two components:

$$|U(t)\rangle = \exp\left\{-\frac{i}{\hbar}\int_0^t \epsilon_{\vec{k}(t')}dt'\right\}\exp\{i\gamma(t)\}\,|u_{\vec{k}(t)}\rangle$$

dynamical time evolution

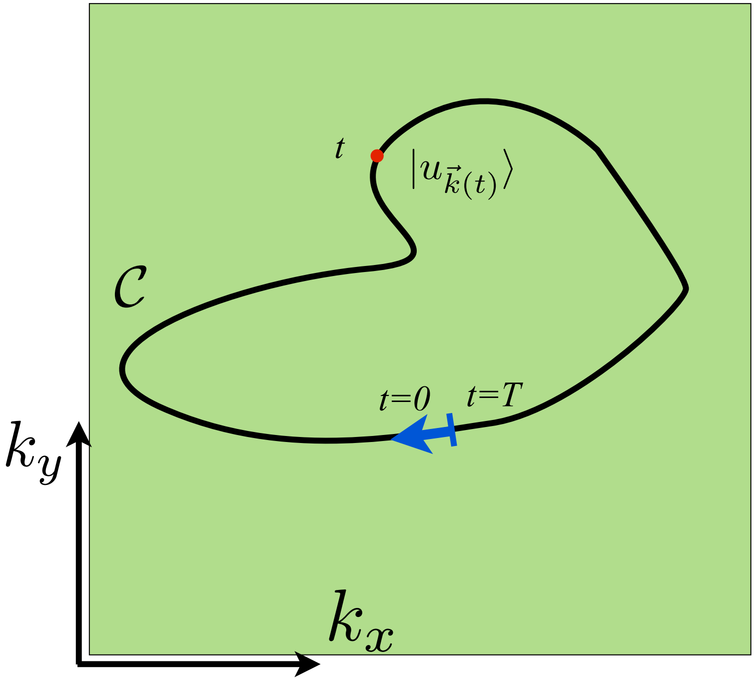
'twist'

Calculating the Berry phase

Michael Berry (1984)

Calculate how wavefunction evolves while moving adiabatically through curve *C* : ***k****(t), t=0...T*

Local basis $\tilde{\mathcal{H}}|u_{\vec{k}}\rangle = \epsilon_{\vec{k}}|u_{\vec{k}}\rangle$



Phase evolution has two components:

$$|U(t)\rangle = \exp\left\{-\frac{i}{\hbar}\int_0^t \epsilon_{\vec{k}(t')}dt'\right\}\exp\left\{i\gamma(t)\right\}|u_{\vec{k}(t)}\rangle$$

Substitute into Schrödinger equation:

$$\tilde{\mathcal{H}}|U(t)\rangle = i\hbar\frac{d}{dt}|U(t)\rangle$$

take

$$\int_0^T dt\langle U(t)| \;\Big|\; \epsilon_{\vec{k}(t)}|U(t)\rangle = i\hbar\left(-\frac{i}{\hbar}\epsilon_{\vec{k}(t)} + i\frac{d}{dt}\gamma(t) + \frac{d\vec{k}}{dt}\frac{d}{d\vec{k}}\right)|U(t)\rangle$$

Berry phase: $$\gamma(\mathcal{C}) = i\int_{\mathcal{C}}\langle u_{\vec{k}}|\frac{d}{d\vec{k}}|u_{\vec{k}}\rangle d\vec{k}$$ purely geometrical!

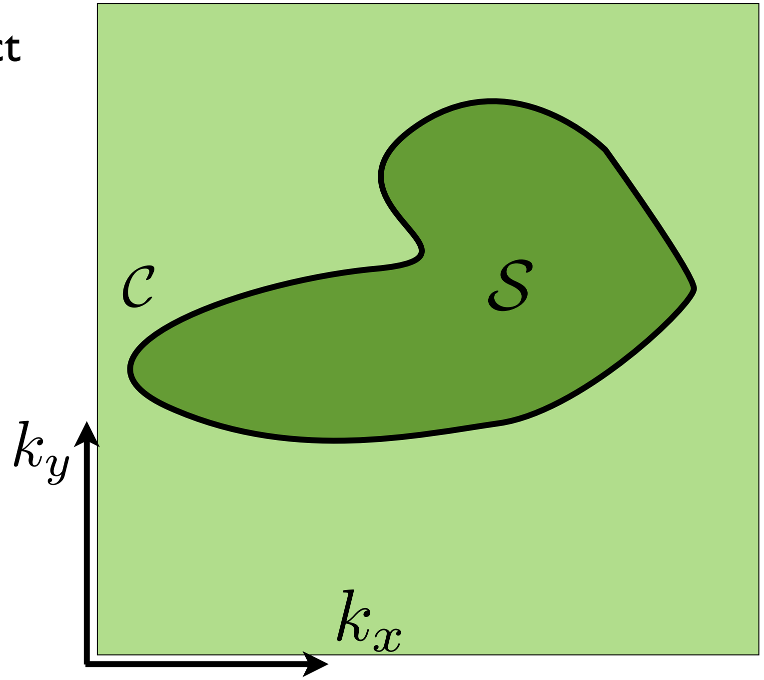
Berry curvature and Chern number

Geometrical phase analogous to Aharonov-Bohm effect

$$\gamma(\mathcal{C}) = i \int_{\mathcal{C}} \langle u_{\vec{k}} | \frac{d}{d\vec{k}} | u_{\vec{k}} \rangle d\vec{k} \equiv \int_{\mathcal{C}} \vec{\mathcal{A}}(\vec{k}) d\vec{k}$$

Effective 'vector potential' called *Berry connection*

$$\vec{\mathcal{A}}(\vec{k}) = i \int_{\mathrm{UC}} u_{\vec{k}}(\vec{r})^* \vec{\nabla}_k u_{\vec{k}}(\vec{r})\, d^2 r$$



Using Stokes' theorem:

$$\gamma(\mathcal{C}) = \int_{\mathcal{C}} \vec{\mathcal{A}}(\vec{k}) d\vec{k} = \int_{\mathcal{S}} \vec{\nabla}_k \times \vec{\mathcal{A}}(\vec{k}) d\vec{\sigma}$$

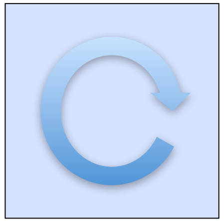
$\mathcal{C} = \partial\mathcal{S}$

Berry curvature: $\vec{\mathcal{B}} = \vec{\nabla}_k \times \vec{\mathcal{A}}(\vec{k})$ ⟸ is a property of the band eigenfunctions, only!

Chern number: $C = \frac{1}{2\pi} \int_{BZ} d^2\mathbf{k}\, \mathcal{B}(\mathbf{k})$ ⟸ takes only integer values!

Evaluating: Berry curvature for Landau-Levels

Choosing the Landau-gauge, i.e. vector potential $\vec{A} = Bx\vec{e}_y$



$$2\pi\ell_0^2 = \frac{h}{eB}$$ spatial extent of of one LL state

Berry connection $\vec{\mathcal{A}}(\vec{k}) = -k_y \ell_0^2 \vec{e}_x$ (this is gauge dependent)

Berry curvature: $\vec{\mathcal{B}} = \ell_0^2 \vec{e}_z$ constant curvature - reflects constant magnetic field

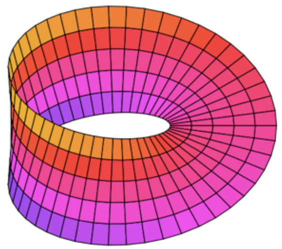
Chern number: $C = 1$

$\Downarrow$

Directly related to quantised Hall conductance:

$$\sigma_{xy} = \frac{e^2}{h} C$$

Like twist in Möbius strip:
Chern number does not vary under small perturbations:
'*Topological invariant*'



Emulating the effect of magnetic fields

Non-zero Berry curvature is not related specifically to magnetic fields only:

$$\vec{\mathcal{A}}(\vec{k}) = i \int_{\mathrm{UC}} u_{\vec{k}}(\vec{r})^* \vec{\nabla}_k u_{\vec{k}}(\vec{r})\, d^2 r$$

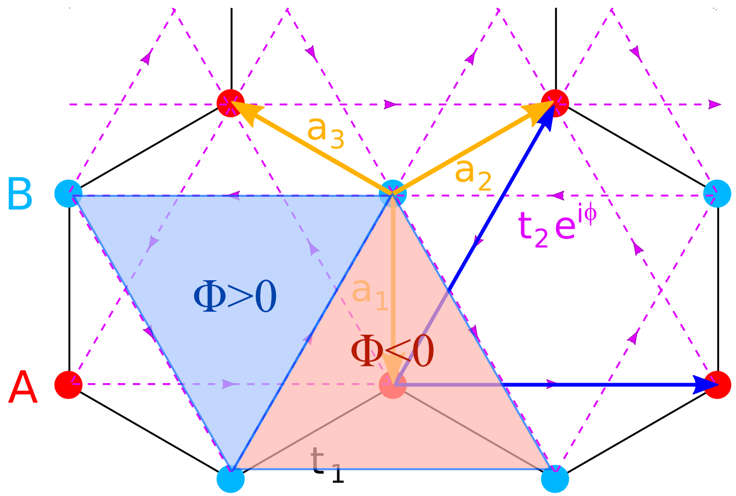
spatial dependency = physical implementation integrated out!

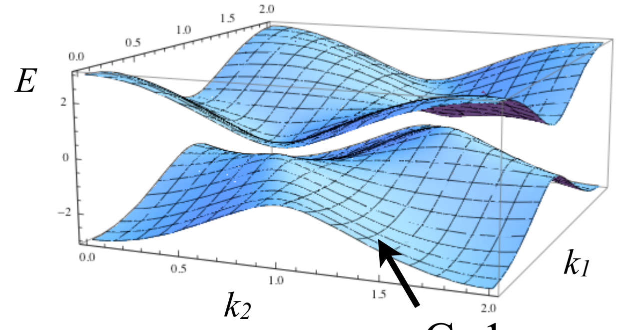
- Other systems with Chern number *C=1* can give rise to a quantized Hall effect

F. D. M. Haldane (1988)

Haldane's Model

F. D. M. Haldane (1988)





$$\mathcal{H} = -\,t_1 \sum_{\langle \mathbf{rr'} \rangle} \left(\hat{a}^\dagger_{\mathbf{r}} \hat{a}_{\mathbf{r}} + h.c.\right) - t_2 \sum_{\langle\langle \mathbf{rr'} \rangle\rangle} \left(\hat{a}^\dagger_{\mathbf{r}} \hat{a}_{\mathbf{r}} e^{i\phi_{\mathbf{rr'}}} + h.c.\right)$$

- Realization of such models might be possible thanks to spin-orbit coupling

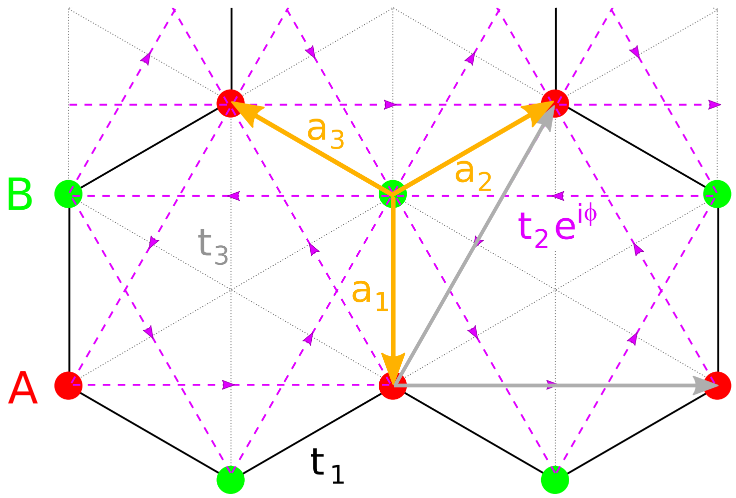
Kane (2005)

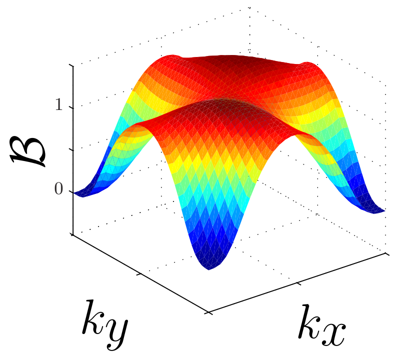
- Proposal: for *flat* bands with $|C|>0$, there may even be a fractional quantum Hall effect!

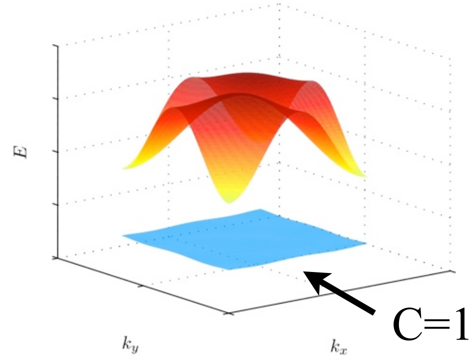
Tang et al. + Neupert et al. + Sun et al. in Phys. Rev. Lett. (2011)

Haldane's Model for strong correlations

F. D. M. Haldane (1988)







$$\mathcal{H} = -\,t_1 \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} (\hat{a}^\dagger_{\mathbf{r}} \hat{a}_{\mathbf{r}} + h.c.) - t_2 \sum_{\langle\langle \mathbf{r}\mathbf{r}' \rangle\rangle} (\hat{a}^\dagger_{\mathbf{r}} \hat{a}_{\mathbf{r}} e^{i\phi_{\mathbf{r}\mathbf{r}'}} + h.c.)$$

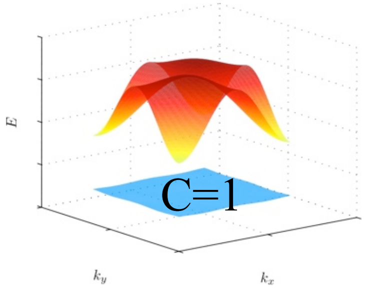
$$-\,t_3 \sum_{\langle\langle\langle \mathbf{r}\mathbf{r}' \rangle\rangle\rangle} (\hat{a}^\dagger_{\mathbf{r}} \hat{a}_{\mathbf{r}} + h.c.)$$

- tight binding model on hexagonal lattice
- no average magnetic flux (but time-reversal symmetry is broken)
- with fine-tuned hopping parameters: obtain flat lower band

$$t_1 = 1,\ t_2 = 0.60,\ t_3 = -0.58 \text{ and } \phi = 0.4\pi$$

Q: Do flat Chern bands yield FQHE physics?

Q: Particles in a nearly flat band with Chern #1 are similar to electrons in a Landau-level, but do repulsive interactions really induce the equivalent of fractional quantum Hall states?



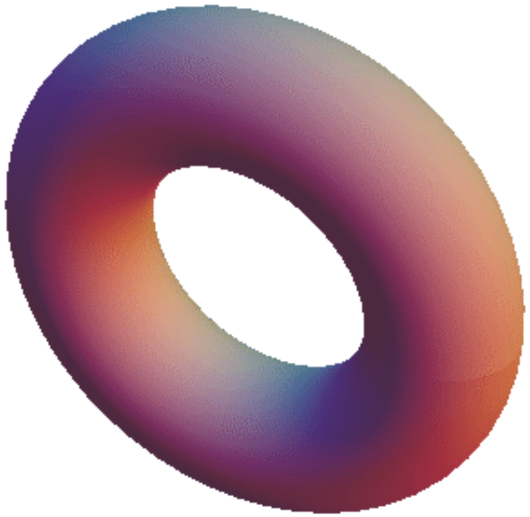
+ Interactions = FQHE ?

- some evidence for such states, which we call "Fractional Chern Insulators" (FCI):
  - existence of a gap & groundstate degeneracy [D. Sheng (2011)]
  - Finite size scaling of gap [Regnault & Bernevig (2011)]
  - count of quasiparticle excitations matches FQHE states (e.g. Laughlin state)

▸ Want to find a new technique that can be used to make robust conclusions about the nature of phases realised in Chern bands

Strategy

- Just like the topology of a geometrical object:



- Topological order is invariant under continuous / adiabatic deformations!

- My approach: Devise a method to deform the wavefunction of a fractional quantum Hall state into a fractional Chern insulator without closing the gap.

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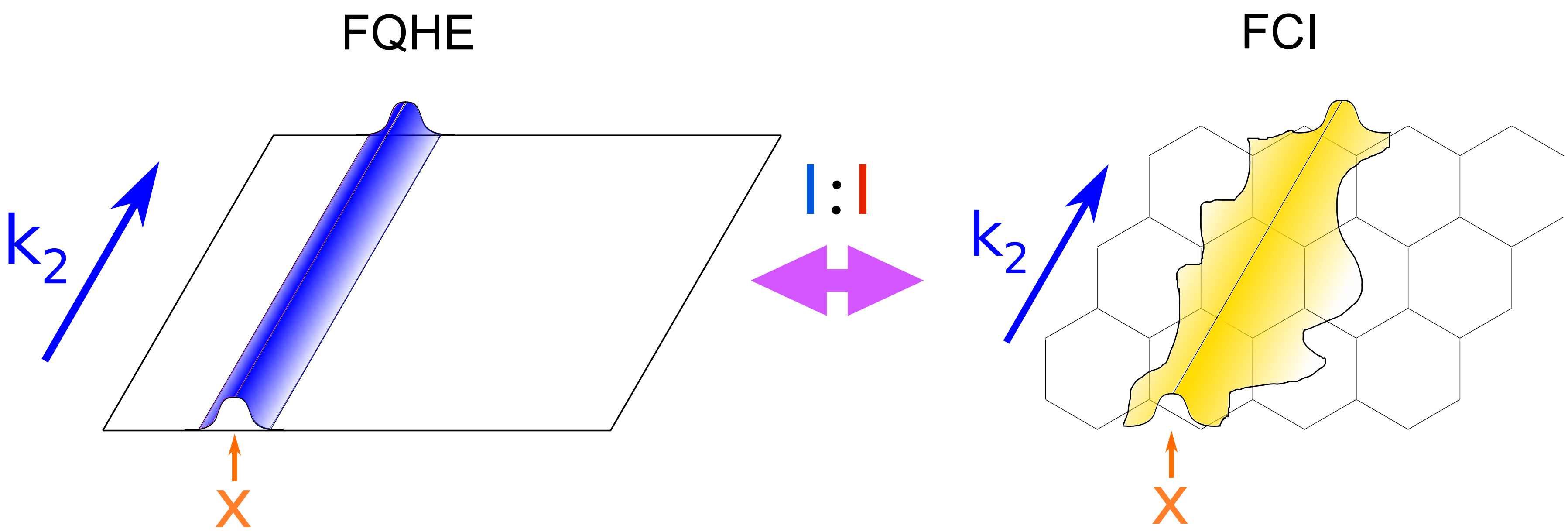
Mapping from FQHE to FCI: Single Particle Orbitals



- Proposal by X.-L. Qi [PRL '11]: Get FCI Wavefunctions by mapping single particle orbitals
- Idea: use Wannier states which are localized in the x-direction
- keep translational invariance in y (cannot create fully localized Wannier state if C>0!)

$$|W(x, k_y)\rangle = \sum_{k_x} f_{k_x}^{(x,k_y)} |k_x, k_y\rangle$$

- Qi's Proposition: using a mapping between the LLL eigenstates (QHE) and localized Wannier states (FCI), we can establish an exact mapping between their many-particle wavefunctions

Wannier states in Chern bands

- construction of a Wannier state at fixed k_y in gauge with $\mathcal{A}_y = 0$

$$|W(x,k_y)\rangle = \frac{\chi(k_y)}{\sqrt{L_x}} \sum_{k_x} e^{-i\int_0^{k_x} \mathcal{A}_x(p_x,k_y)dp_x} \times e^{ik_x \frac{\theta(k_y)}{2\pi}} \times e^{-ik_x x}|k_x,k_y\rangle$$

'Parallel transport' of phase — Berry connection indicates change of phase due to displacement in BZ

'Polarization' — ensures periodicity of WF in $k_y \rightarrow k_y + 2\pi$

Fourier transform

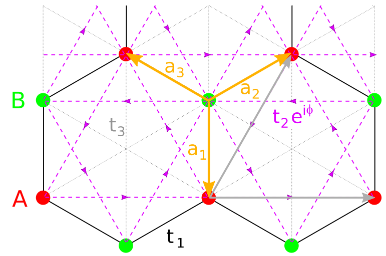
- or, more simply we can think of the Wannier states as the eigenstates of the position operator

$$\hat{X}^{cg} = \lim_{q_x\to 0} \frac{1}{i}\frac{\partial}{\partial q_x}\bar{\rho}_{q_x} \quad \Rightarrow \quad \hat{X}^{cg}|W(x,k_y)\rangle = [x - \theta(k_y)/2\pi]|W(x,k_y)\rangle$$

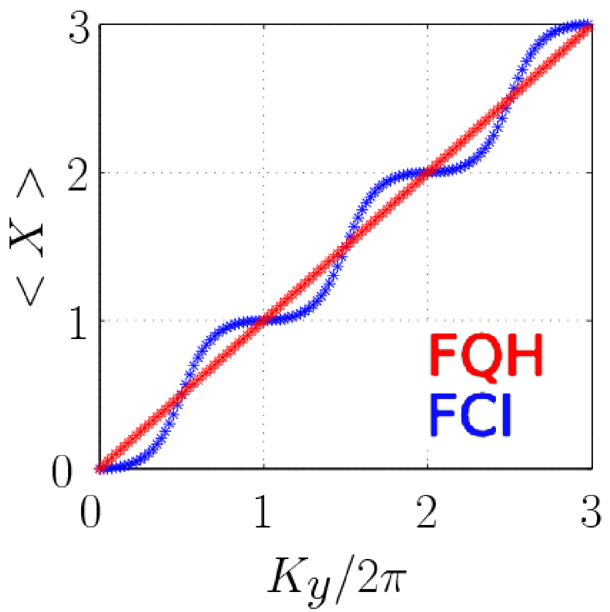
- role of polarization: displacement of centre of mass of the Wannier state

$$\theta(k_y) = \int_0^{2\pi} \mathcal{A}_x(p_x,k_y)dp_x$$

Associating Quantum Numbers



- The Wannier basis for the Chern band produces a sequence of single-particle states with monotonously increasing position
- Similar to the linearly increasing position of the Landau-level basis



$$k_y = 2\pi n_y / L_y$$

$$K_y = k_y + 2\pi x = 2\pi j / L_y$$

$$j = n_y + L_y x = 0, 1, ..., N_\phi - 1$$

- Formulate degenerate perturbation theory in this basis for a fractionally filled Chern band

$$\hat{\mathcal{H}} = K.E. + \sum_{i<j} \hat{V}(\vec{r}_i - \vec{r}_j) \qquad \mathcal{H} = \begin{pmatrix} V_{11} & V_{12} & V_{13} & \dots \\ V_{21} & V_{22} & V_{23} & \dots \\ V_{31} & V_{32} & \ddots & \\ \vdots & \vdots & & \ddots \end{pmatrix}$$

many-body basis

with $V_{ij} = \langle \alpha_j | \hat{V} | \alpha_i \rangle$

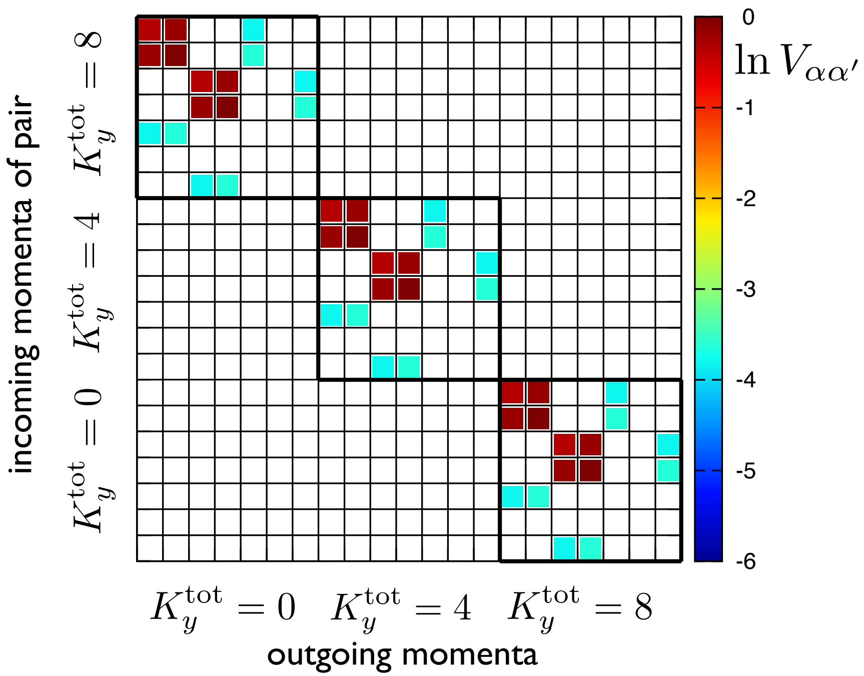
Visualising contact interactions for bosons

Th. Scaffidi & GM, Phys. Rev. Lett. (2012)

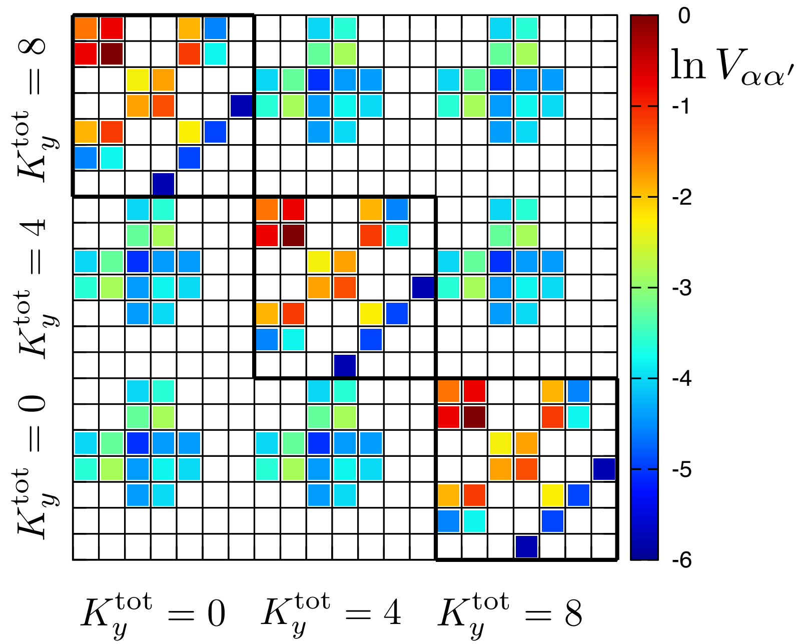
- Magnitude of two-body matrix elements for delta interactions in the Haldane model $\mathcal{H} = \begin{pmatrix} V_{11} & V_{12} & V_{13} & \dots \\ V_{21} & V_{22} & V_{23} & \dots \\ V_{31} & V_{32} & \ddots & \\ \vdots & \vdots & & \ddots \end{pmatrix}$

$$V(\vec{r}_i - \vec{r}_j) \propto \delta(\vec{r}_i - \vec{r}_j)$$

FQHE



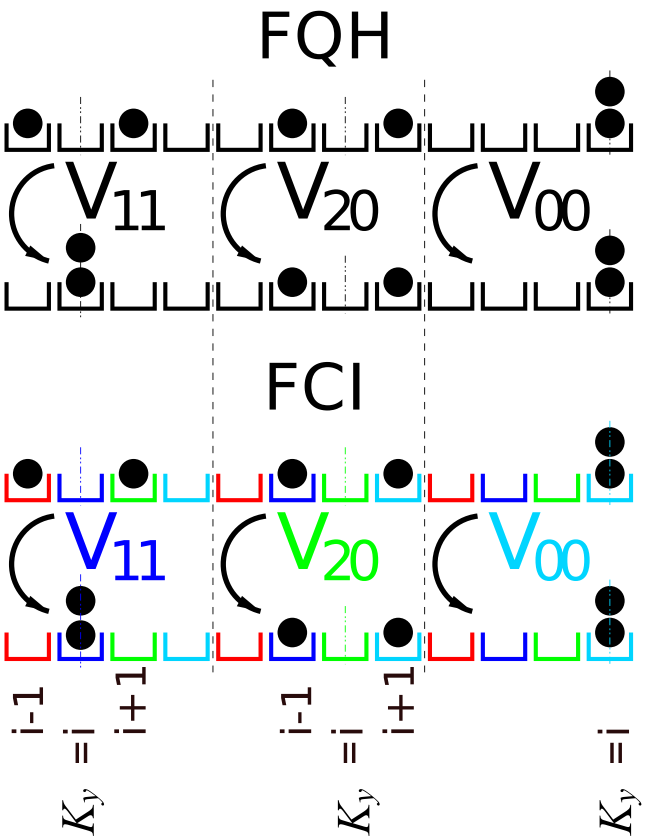
FCI

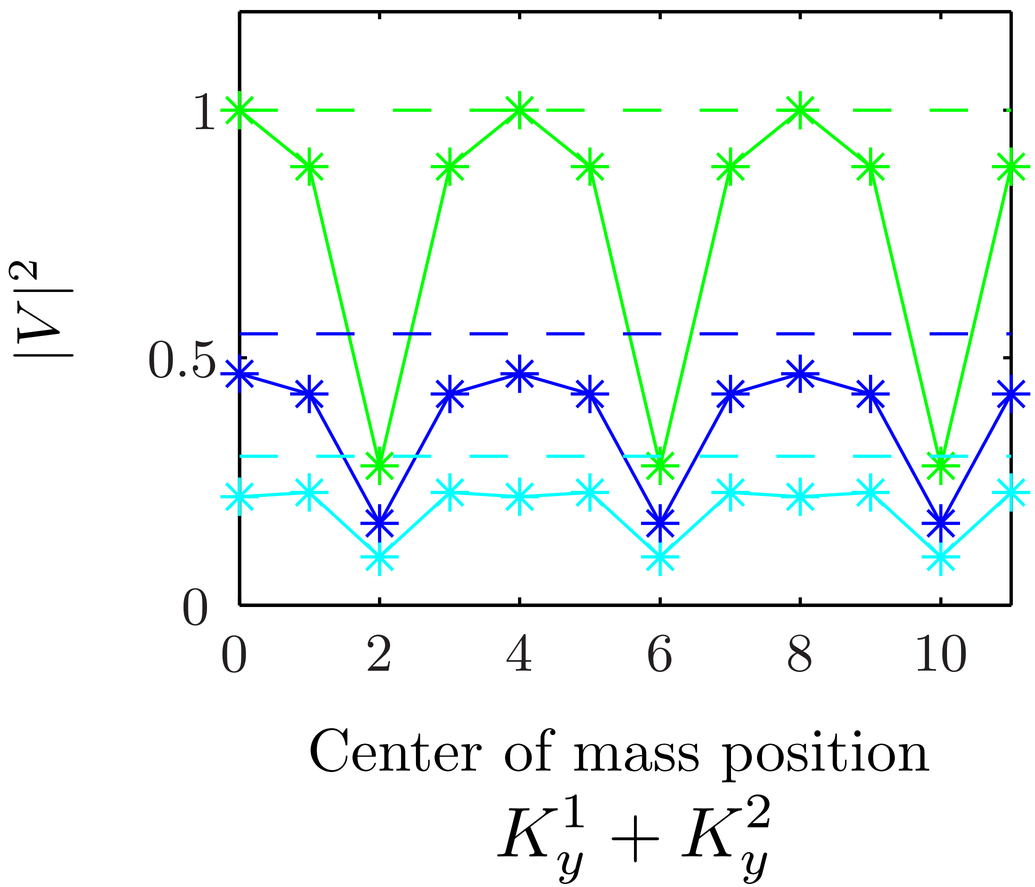


- System shown: two-body interactions for $L_x \times L_y = 3 \times 4$
- Matrix elements differ in magnitude, but overall similarities are present
- Different block-structure due to non-conservation of linearised momentum K_y

Reduced translational invariance in K_y

- A closer look at some short range hopping processes





- for FCI: hopping amplitudes depend on position of centre of mass / K_y

Interpolating in the Wannier basis

- Can write both states in single Hilbert space with the same overall structure (indexed by K_y) and study the low-lying spectrum numerically (exact diagonalization)
- Can study adiabatic deformations from the FQHE to a fractionally filled Chern band

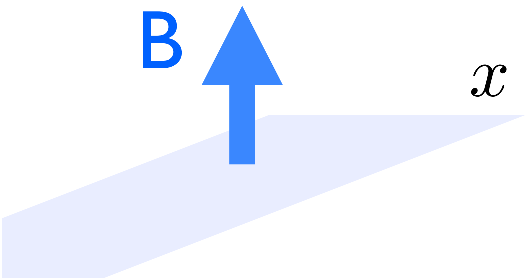
$$\mathcal{H}(x) = \frac{\Delta_{\rm FCI}}{\Delta_{\rm FQHE}}(1-x)\mathcal{H}^{\rm FQHE} + x\mathcal{H}^{\rm FCI}$$

- Here: look at half-filled band for bosons

FQHE of Bosons at $\nu = 1/2$

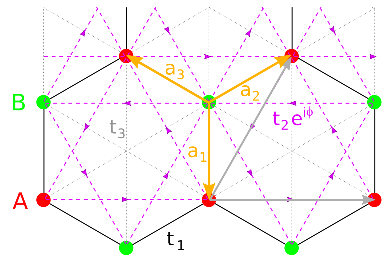
Half filled band of the (flattened) Haldane-model

Laughlin state ⟷ Same topological phase?



$x = 0$

$x = 1$



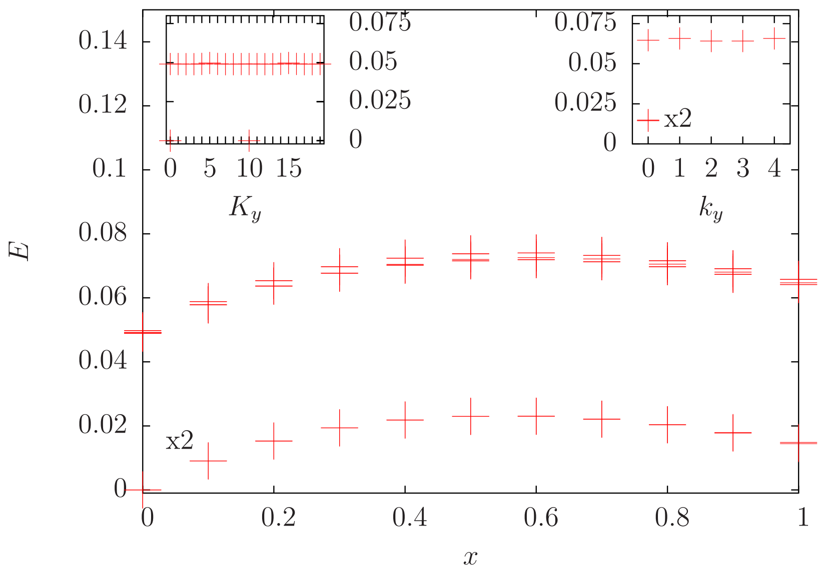
Th. Scaffidi & GM, Phys. Rev. Lett. (2012)

Adiabatic continuation in the Wannier basis

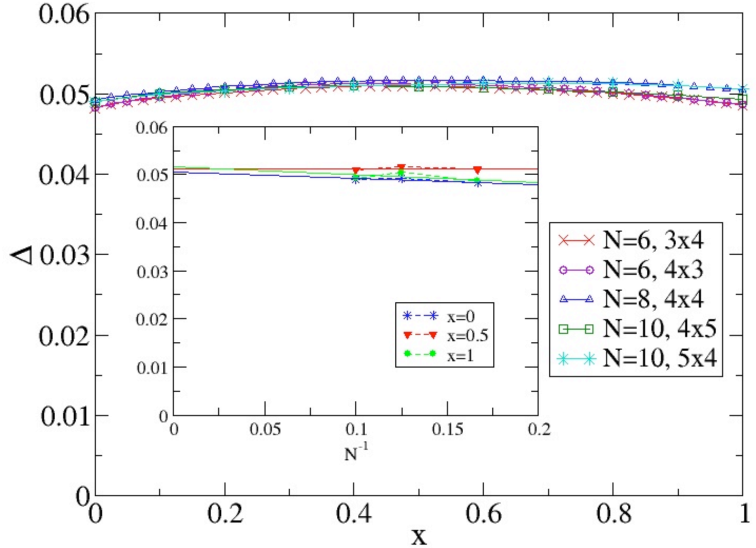
- Spectrum for N=10
(Hilbert space of dimension d=5x10[6]):

- Gap for different system sizes & aspect ratios:

FQHE FCI



FQHE FCI



- The gap remains open for all x!
- We confirm the Laughlin state is adiabatically connected to the groundstate of the half-filled topological flat band of the Haldane model

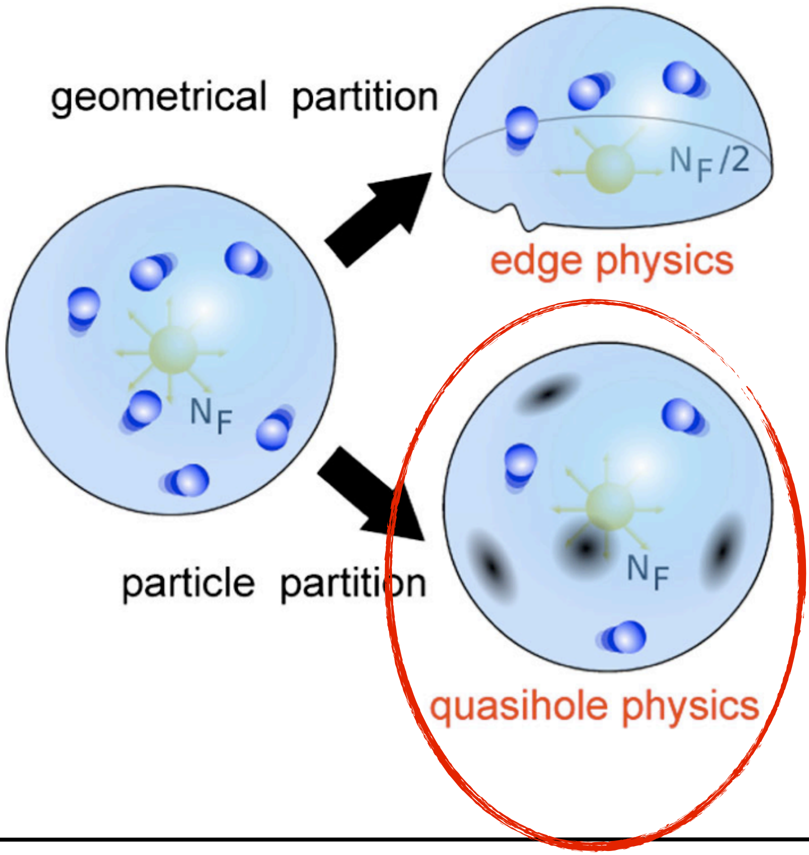
▸General strategy with possibility to test & predict topological order in the thermodynamic limit

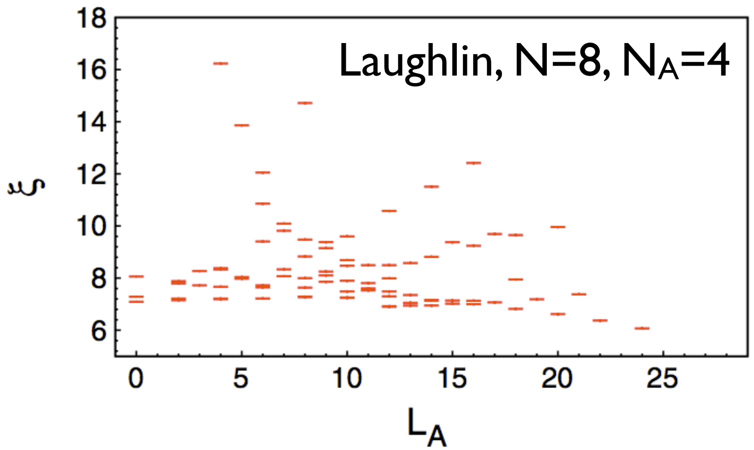
Th. Scaffidi & GM, Phys. Rev. Lett. (2012)

Entanglement spectra and quasiparticle excitations

• Entanglement spectrum: arises from Schmidt decomposition of ground state into two groups A, B
=> Schmidt eigenvalues ξ plotted over quantum numbers for symmetries within each block

$$|\Psi\rangle = \sum_{\varpi} \sum_{i} e^{-\xi_{\varpi,i}/2} |\Psi^{A}_{\varpi,i}\rangle \otimes |\Psi^{B}_{\varpi,i}\rangle$$





Dominant (universal) eigenvalues of PES yield count of excited states - and their wavefunctions - from groundstate wavefunction only!

credit: Sterdyniak et al. PRL 2011

FCI: Adiabatic continuation of the entanglement spectrum

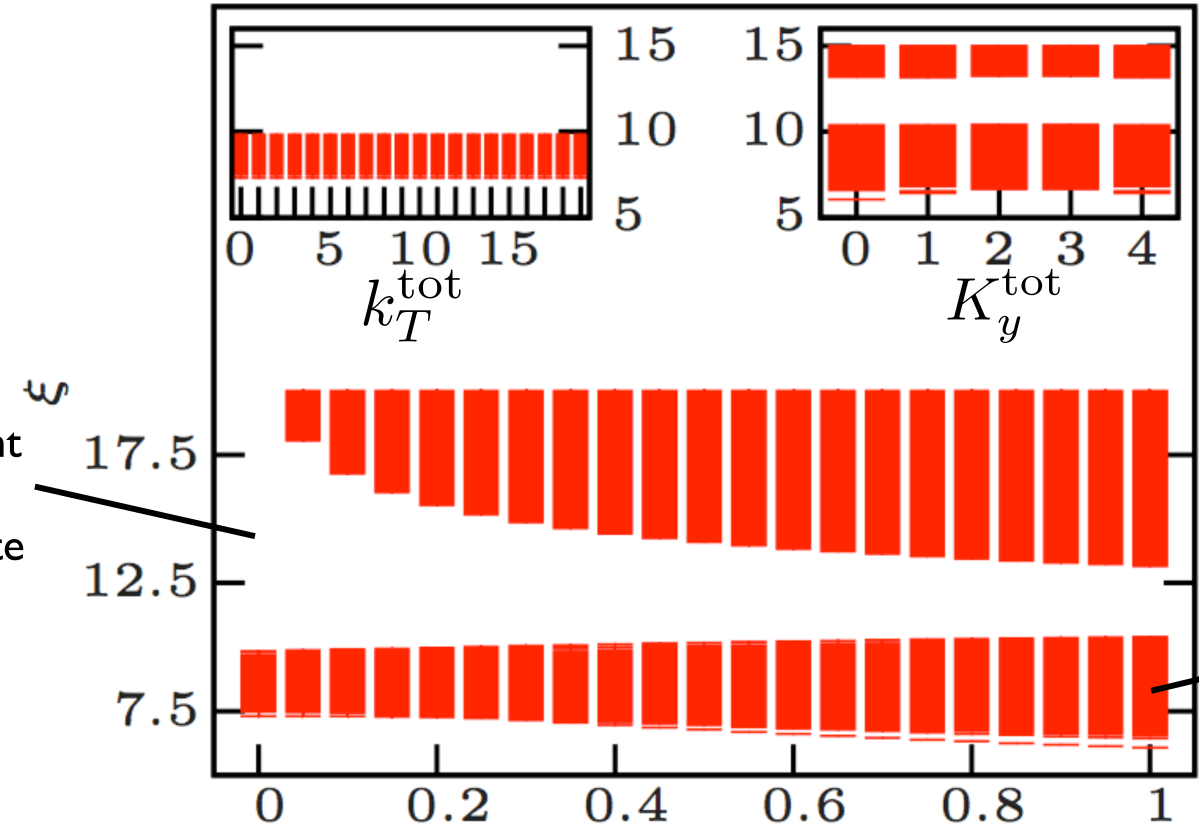
Total #eigenvalues below entanglement gap = 4x(201 + 200 + 200 + 200 + 200)

Total #eigenvalues below entanglement gap = 804 + 800 + 800 + 800 + 800

$N = 10$

$N_A = 5$

'Infinite' entanglement gap for pure Laughlin state

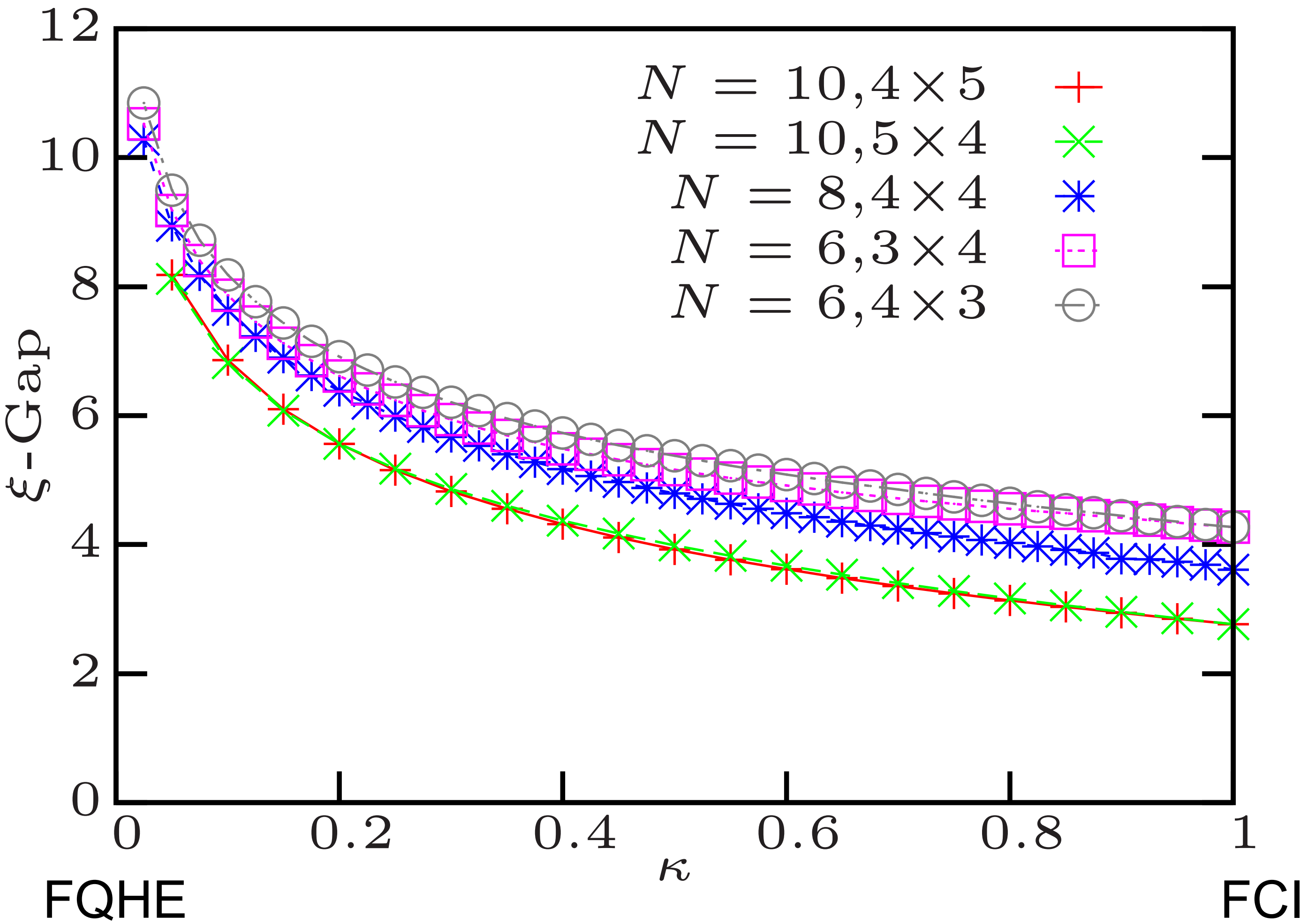


Same number of states for all x

Dictionary: FQHE FCI

$$|\Psi\rangle = \sum_{\varpi} \sum_{i} e^{-\xi_{\varpi,i}/2} |\Psi^A_{\varpi,i}\rangle \otimes |\Psi^B_{\varpi,i}\rangle$$

Finite size behaviour of entanglement gap



- The entanglement gap remains open for all values of the interpolation parameter k
- Finite size scaling behaviour encouraging, but analytic dependency on system size unknown

Summary of analytic continuation

Analytic continuation:

- provides a formal proof of identical topological order between two phases

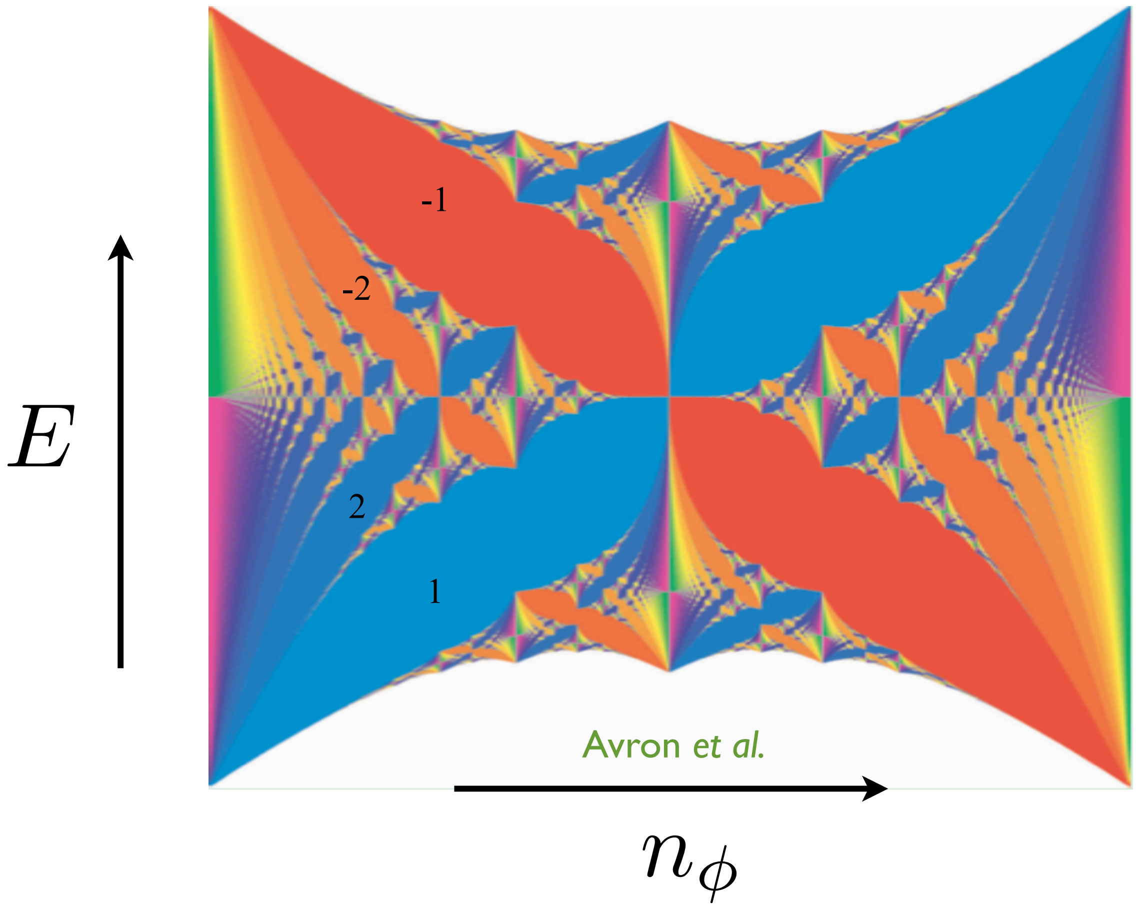
- may allows robust conclusions about the thermodynamic limit from finite size data

- Successfully applied to a range of systems:
  - Haldane model: Laughlin state (Scaffidi & Möller, PRL 2012)
  - Kagomé lattice model: Laughlin & Moore-Read states (Liu & Bergholtz, PRB 2013)

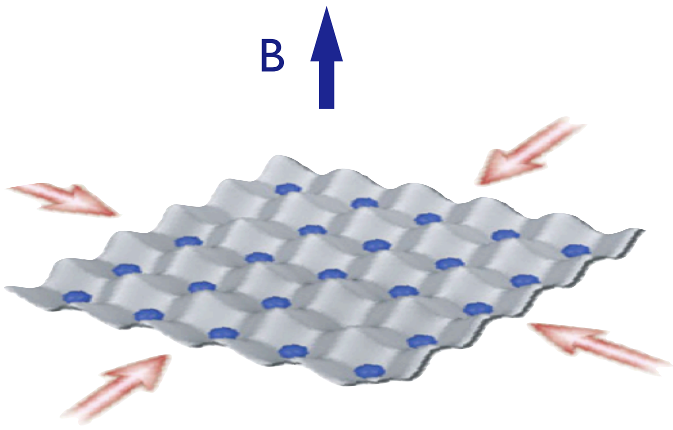
- Possibility to simulate quantum Hall physics using spin-orbit coupling in solid state materials confirmed.

Is there new physics in Chern Insulators?

- clearly YES: e.g., higher Chern-numbers
- characteristic system featuring bands of any Chern number: the square lattice with constant magnetic flux



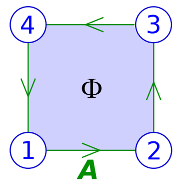
color-coding of gaps by Chern numbers - blue: positive, red: negative integers



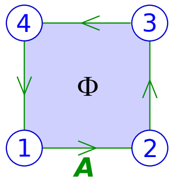
- use Aharonov-Bohm effect to emulate flux

$$\mathcal{H}_c = -J \sum_{\langle \alpha,\beta \rangle} \left[\hat{b}_\alpha^\dagger \hat{b}_\beta e^{iA_{\alpha\beta}} + h.c. \right]$$

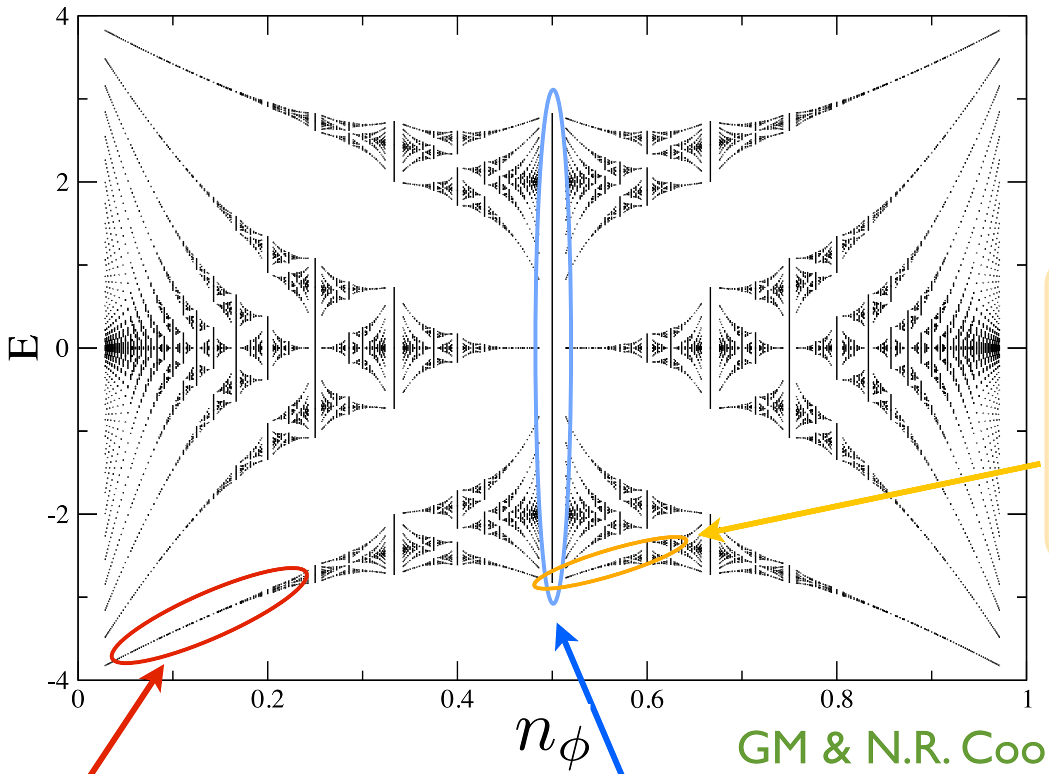
$$\sum_{\square} A_{\alpha\beta} = 2\pi n_\phi$$



Many-body phases of the Hofstadter spectrum



square lattice at flux density n_ϕ



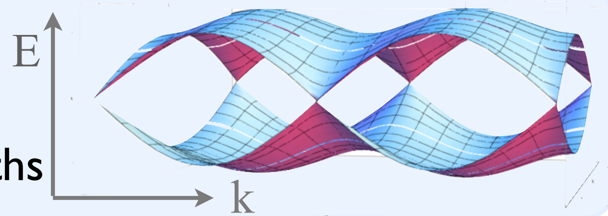
narrow bands with a gap
➜ novel Fractional Quantum Hall states beyond those known in the continuum!

GM & N.R. Cooper, PRL 2009; PRA 2010

narrow band (lowest Landau level)
➜ Fractional Quantum Hall states

A. Sørensen et al., PRL 2005

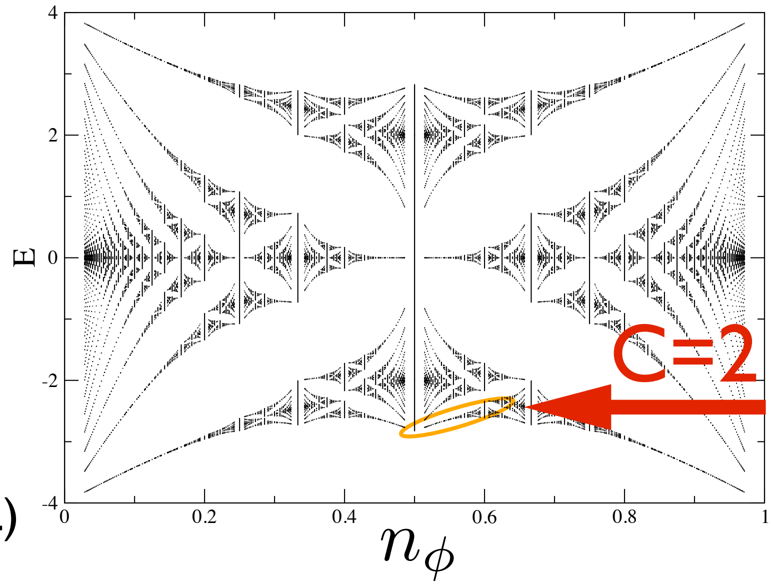
$n_\phi=1/2$: wide bands
➜ condensates at all interaction strengths



Strongly correlated phases in the Hofstadter hierarchy

Interacting Bose-Hubbard model with flux n_ϕ

$$\mathcal{H} = -J \sum_{\langle\alpha,\beta\rangle} \left[\hat{b}_\alpha^\dagger \hat{b}_\beta e^{iA_{\alpha\beta}} + h.c.\right] + \frac{1}{2}U \sum_\alpha \hat{n}_\alpha(\hat{n}_\alpha - 1) - \mu \sum_\alpha \hat{n}_\alpha$$



A) $n_\phi = \frac{1}{2} + \epsilon$ and limit of small U

physics ~ QHE in Landau levels with subband index (C=2)

Halperin states: R. Palmer & D. Jaksch, PRL 2006, PRA 2008

Double Pfaffian (paired) state: L. Hormozi, GM & S.H. Simon, PRL 2012

B) hardcore limit $U \to \infty$: physics of composite fermions

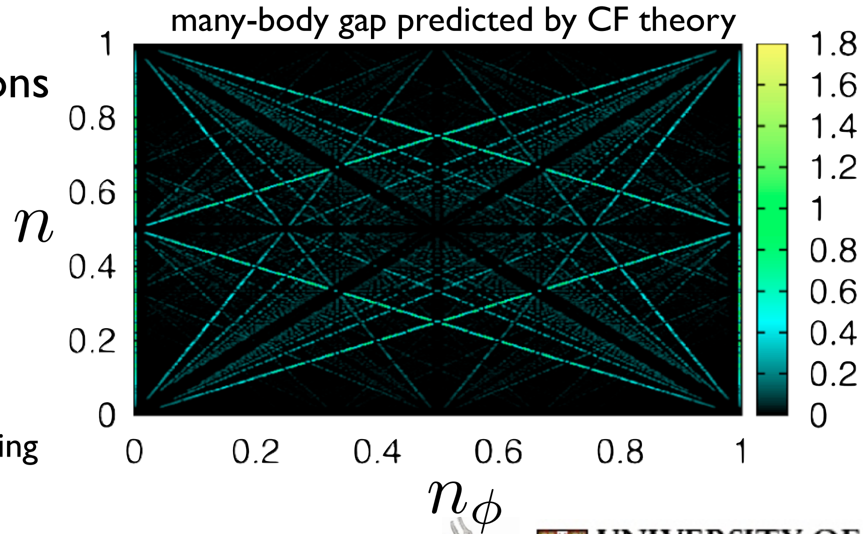
Interactions favour flux attachment, suppressing any double occupation at expense of kinetic energy

$$\Psi_\mathrm{B}(\{\mathbf{r}_i\}) \propto \underbrace{\psi_\mathrm{J}^{(\phi_x,\phi_y)}(\{\mathbf{r}_i\})}_{\text{equivalent of a Jastrow-factor}} \; \psi_\mathrm{CF}^{(-\phi_x,-\phi_y)}(\{\mathbf{r}_i\})$$

equivalent of a Jastrow-factor

Composite Fermions filling Hofstadter bands

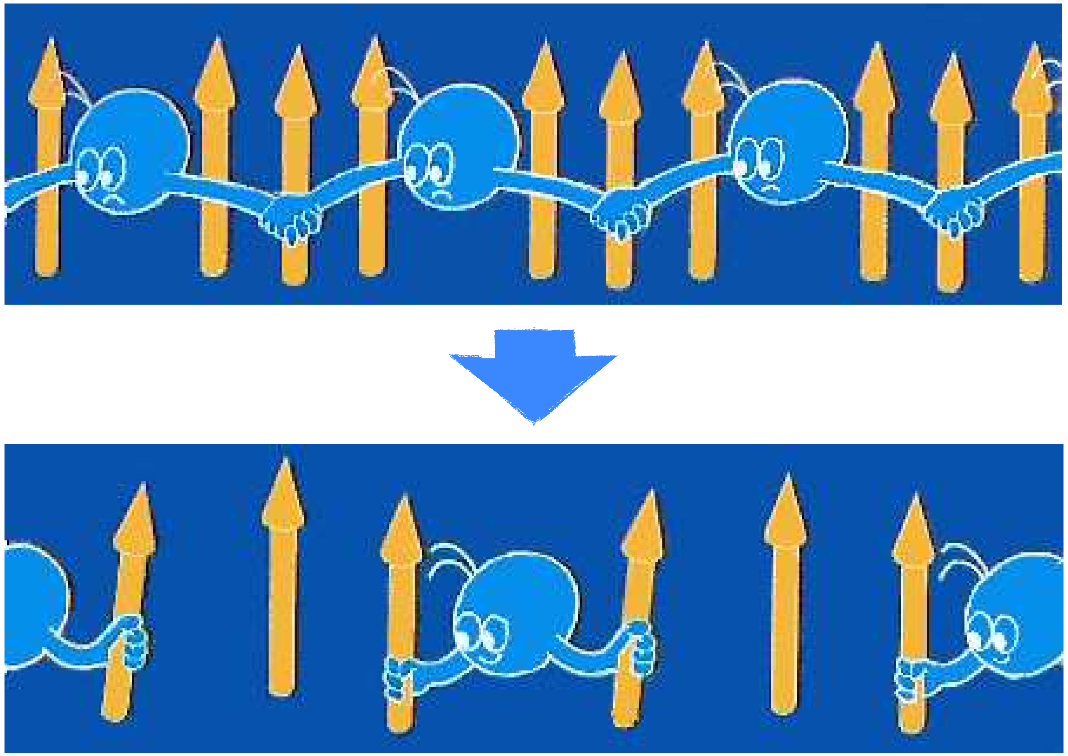
GM & N.R. Cooper, PRL 2009



The Composite Fermion Approach

$$\mathcal{H} = -J \sum_{\langle \alpha, \beta \rangle} \left[\hat{b}_\alpha^\dagger \hat{b}_\beta e^{iA_{\alpha\beta}} + h.c. \right] + \frac{1}{2} U \sum_\alpha \hat{n}_\alpha (\hat{n}_\alpha - 1) - \mu \sum_\alpha \hat{n}_\alpha$$

Account for repulsive interactions $U>0$ by “flux-attachment” (Fradkin 1988, Jain 1989)



Continuum Landau-level for fermions at filling 1/3: three flux per particle

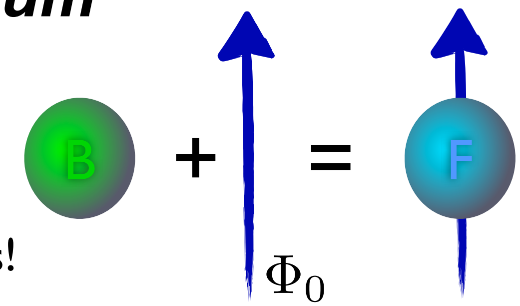
Composite fermions = electron + 2 flux quanta

$$\Psi \propto \prod_{i<j} (z_i - z_j)^2 \Psi_{\mathrm{CF}}$$

1 flux per composite particle

drawings: K. Park

Composite Fermions in the Hofstadter Spectrum



1. Flux attachment for bosonic atoms: $n_\phi^* = n_\phi \mp n$

$$\Psi_B \propto \prod_{i<j}(z_i - z_j)\Psi_{\rm CF}$$ ⇒ transformation of statistics!

2. Effective spectrum at flux n_ϕ^* is again a Hofstadter problem

⇒ weakly interacting CF will fill bands, so obtain density n by counting bands using fractal structure

⇒ linear relation of flux and density for bands under a gap

$$n = \alpha n_\phi^* + \delta$$

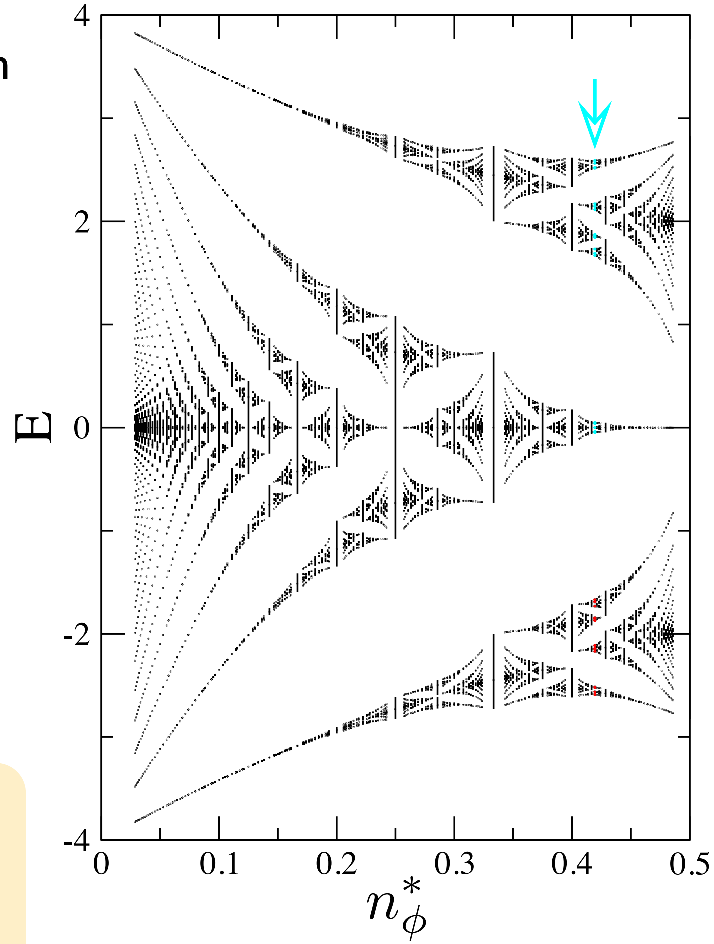
3. Construct Composite Fermion wavefunction

continuum: $$\Psi_{\rm B}(\{\mathbf{r}_i\}) \propto \mathcal{P}_{LLL} \underbrace{\prod_{i<j}(z_i - z_j)}\ \psi_{\rm CF}(\{\mathbf{r}_i\})$$

Vandermonde / Slater determinant of LLL states

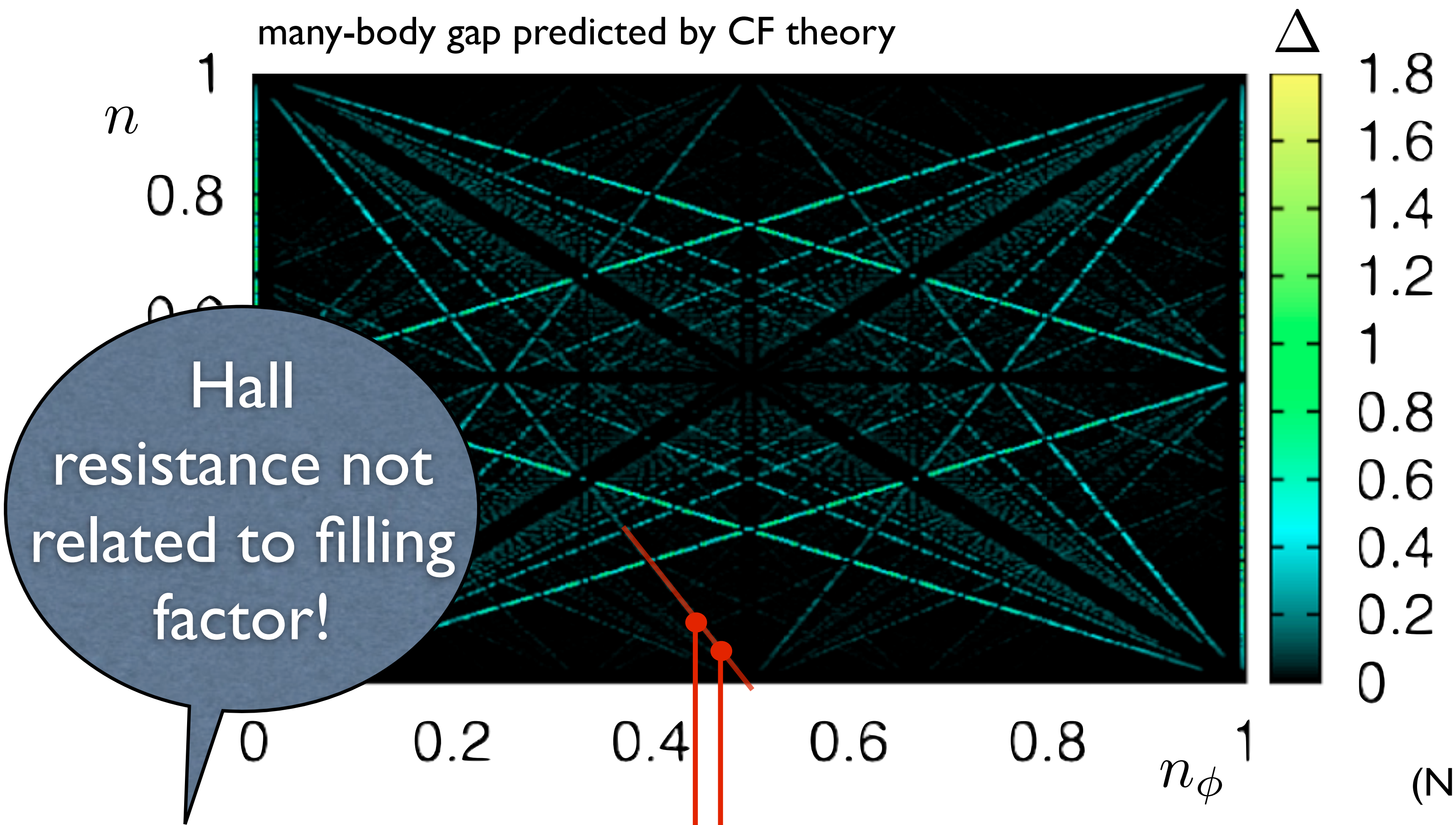
lattice: $$\Psi_{\rm B}(\{\mathbf{r}_i\}) \propto \underbrace{\psi_{\rm J}^{(\phi_x,\phi_y)}(\{\mathbf{r}_i\})}\ \psi_{\rm CF}^{(-\phi_x,-\phi_y)}(\{\mathbf{r}_i\})$$

Slater determinant of Hofstadter orbitals at flux density $n_\phi^0 = n$



GM & N. R. Cooper, PRL 2009

Composite Fermion Theory: Predictions & Verification



(N=5 particles)

testing CF states numerically at

$$n = 1 - 2n_\phi$$

$$n = 1/9\text{: } \mathcal{O} = |\langle \Psi_{CF} | GS \rangle|^2 \simeq 0.46$$

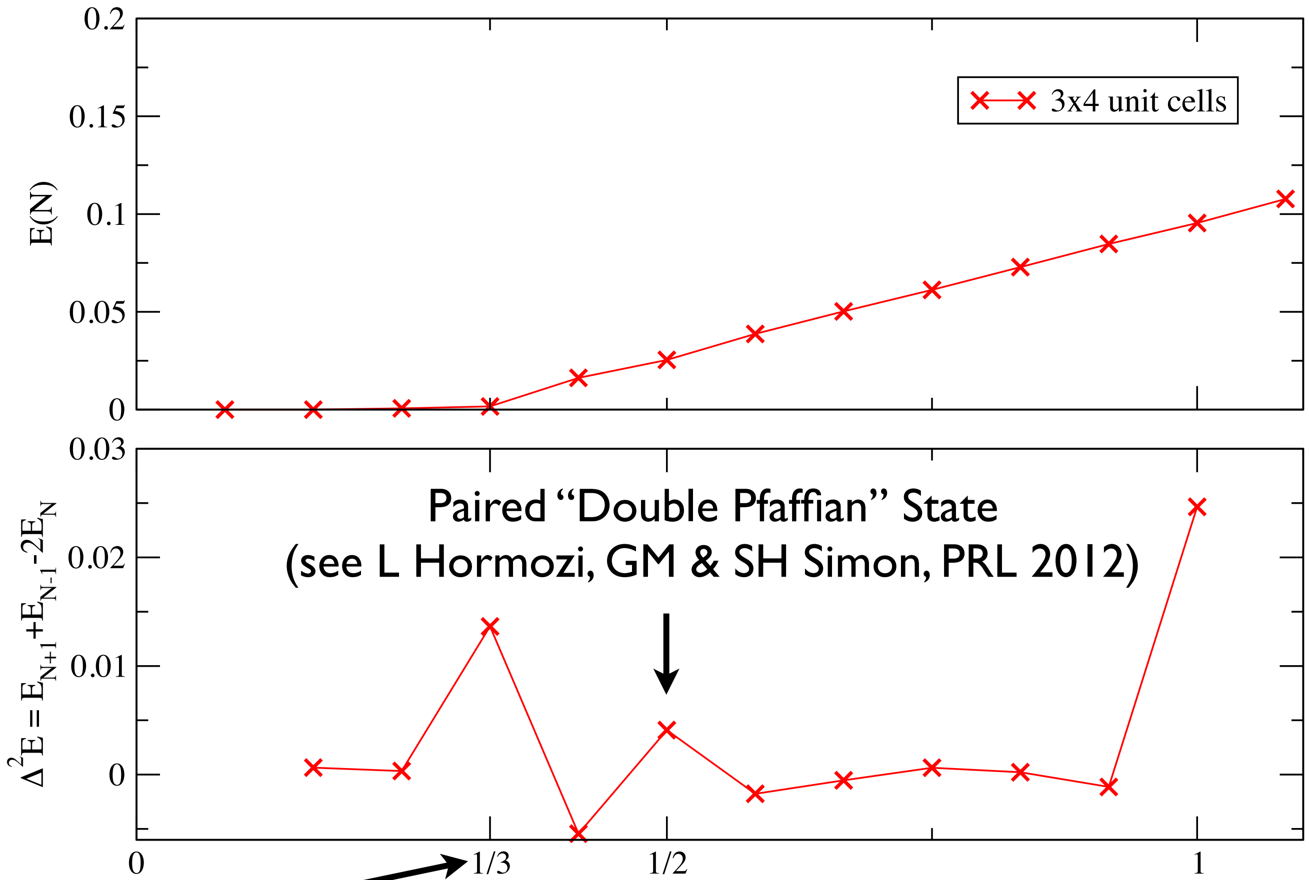
$$n = 1/7\text{: } \mathcal{O} = |\langle \Psi_{CF} | GS \rangle|^2 \simeq 0.56$$

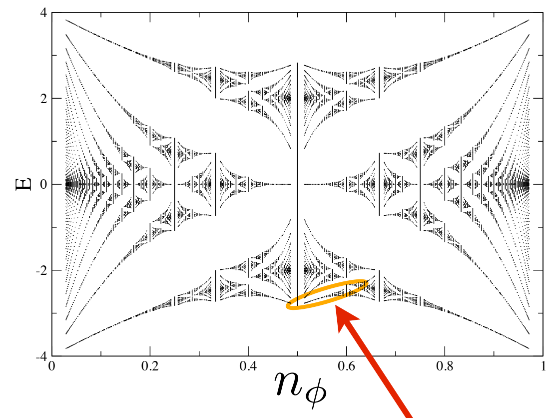
⇒ *unprojected* CF state captures overall structure of correlations well (d=1.2M for result at n=1/9)

Composite Fermion States in the band projected model

take projection to the Chern-# C=2 subband dominating the $n = 1 - 2n_\phi$ CF states

$n_\phi = 4/9$ for bosons, using 3x4 *k*-points (N_s=12), magnetic unit cell of 3x3 sites





C=2

composite fermion states with both positive and negative flux attachment seen - and a paired state, too!

CF state at positive flux attachment

$$\tilde{\nu} = \frac{n}{1 - 2n_\phi}$$

CF state at negative flux attachment

Conclusions

- Many new systems in which FQHE-like physics of Chern-Insulators can be induced:
  - spin-orbit coupled materials
  - optical flux lattices or artifical gauge fields in cold atomic gases

- For C=1 bands, can identify quantum liquids in FCI models by analytic continuation to usual FQHE in the continuum lowest Landau level - useful tool for identifying phases
T. Scaffidi & GM, Phys. Rev. Lett. 109, 246805 (2012).

- New physics can be found in Chern bands with higher $C > 1$
  - can use picture of LL + subband / color index to describe $C > 1$ bands, but color index is not conserved!
  - many-body phases in $C > 1$ bands first analyzed for the Hofstadter spectrum: GM & N. R. Cooper, "Composite Fermion Theory for Bosonic Quantum Hall States on Lattices", Phys. Rev. Lett. 103, 105303 (2009).