$$\delta\phi = \epsilon\left(t\dot{\phi} + z\phi'\right)$$

Noether's theorem argument:

$$\delta\mathcal{L} = \frac{\partial}{\partial z}\left(\frac{\partial \mathcal{L}}{\partial \phi'}\,\delta\phi\right) + \frac{\partial}{\partial t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\,\delta\phi\right)$$

Usually (for a symmetry) $\delta\mathcal{L} = 0$

But here... $\mathcal{L}$ is <u>not</u> invariant under the transformation

Rather: $\mathcal{L} \to b^2\mathcal{L}(bz, bt)$

$$\Rightarrow \quad = (1+\varepsilon)^2\,\mathcal{L}\left((1+\varepsilon)z, (1+\varepsilon)t\right)$$

Expanding to 1st order in ε

$$\Rightarrow \boxed{\delta\mathcal{L} \simeq 2\epsilon\,\mathcal{L} + \varepsilon z\frac{\partial\mathcal{L}}{\partial z} + \varepsilon t\frac{\partial\mathcal{L}}{\partial t}}$$

Hence:

$$2\epsilon\mathcal{L} + \varepsilon z \mathcal{L}' + \varepsilon t \dot{\mathcal{L}} = \frac{\partial}{\partial z}\left(\frac{\partial \mathcal{L}}{\partial \varphi'}\delta\varphi\right) + \frac{\partial}{\partial t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\varphi}}\delta\varphi\right)$$

$$\Rightarrow \frac{\partial}{\partial z}\left(\varepsilon z \mathcal{L}\right) + \frac{\partial}{\partial t}\left(\varepsilon t \mathcal{L}\right) = \frac{\partial}{\partial z}\left(\frac{\partial \mathcal{L}}{\partial \varphi'}\delta\varphi\right) + \frac{\partial}{\partial t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\varphi}}\delta\varphi\right)$$

Usually this L.H.S. is zero ($\delta\mathcal{L}=0$)

Then we interpret this as a continuity equation

$$0 = \frac{\partial}{\partial z}J_z + \frac{\partial}{\partial t}\rho$$

with $J_z = \frac{\partial \mathcal{L}}{\partial \psi'} \delta\psi$ $\quad \rho = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} \delta\psi$

Here the LHS is non zero. However we can still turn it into a continuity equation, as:

$$0 = \frac{\partial}{\partial z}\left(\frac{\partial \mathcal{L}}{\partial \psi'} \delta\psi - \varepsilon z \mathcal{L}\right)$$

$$+ \frac{\partial}{\partial t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\psi}} \delta\psi - \varepsilon t \mathcal{L}\right)$$

From which we find:

$J_z = \frac{\partial \mathcal{L}}{\partial \psi'} \delta\psi - \varepsilon z \mathcal{L}$ $\quad \rho = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} \delta\psi - \varepsilon t \mathcal{L}$

The conserved (global) charge is

$$Q \equiv \int \rho \, dz$$

Hence "Q_2" $= \int \left(\frac{\partial \mathcal{L}}{\partial \dot{\psi}} \delta\psi - \varepsilon t \mathcal{L} \right) dz$

Use $\mathcal{L} = \frac{\pi a^2}{2} \left[\rho \dot{\psi}^2 - K \psi'^2 \right]$

$$\delta\psi = \varepsilon (t \dot{\psi} + z \psi')$$

To get $\frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \frac{\pi a^2}{2} \left[2 \rho \dot{\psi} \right]$

$$Q_2 = \int \frac{\pi a^2}{2} \varepsilon \Big[2 \rho \dot{\psi} (t \dot{\psi} + z \psi')$$
$$- t (\rho \dot{\psi}^2 - K \psi'^2) \Big] \, dz$$

$$\boxed{Q_2 = \frac{\pi a^1}{2} \varepsilon \int \left[\rho t \dot{\psi}^2 + K t \psi'^2 + 2 \rho z \dot{\psi} \psi' \right] dz}$$

+ we can set $\varepsilon = 1$ (or any prefactor to this we want)